Message-passing and traffic optimization on networks

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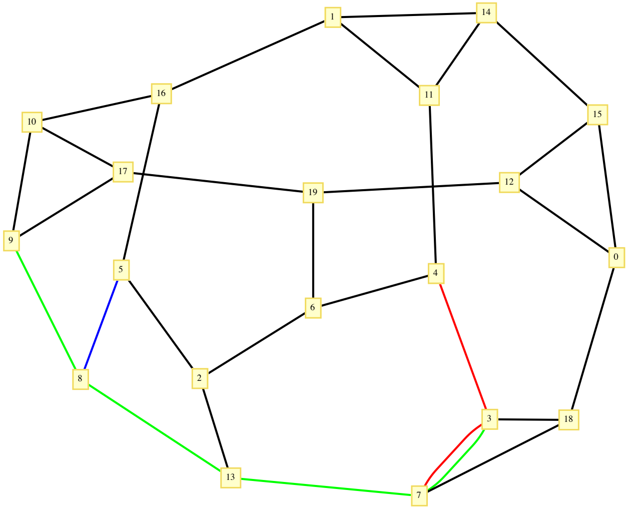
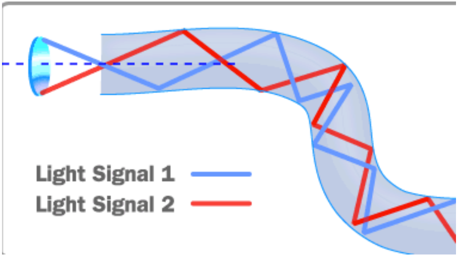
Summary

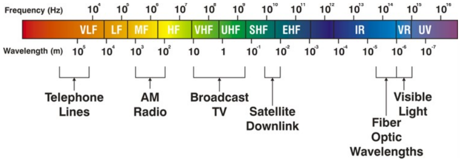
1. Motivation: optical network example
2. Node disjoint path
3. Edge disjoint path
4. Future perspectives

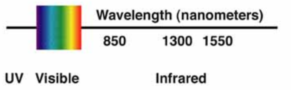
Motivation



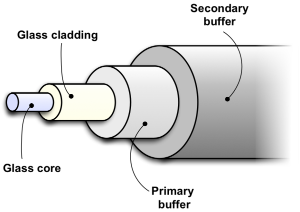
Optical network communication.

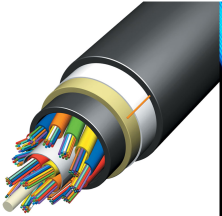


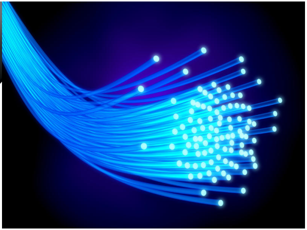




Optical network communication.

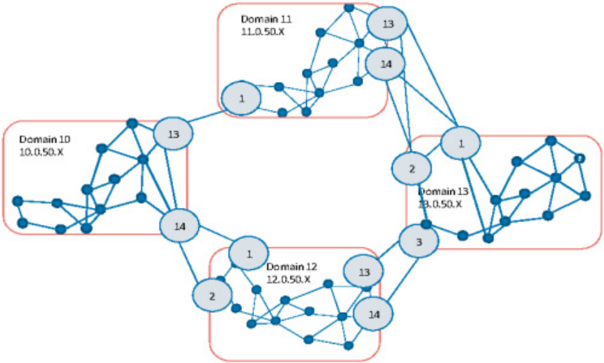






- Low attenuation
- No electromagnetic interference
- High bandwidth

The problem: Routing and Wavelength assignment (RWA)



Constraints:

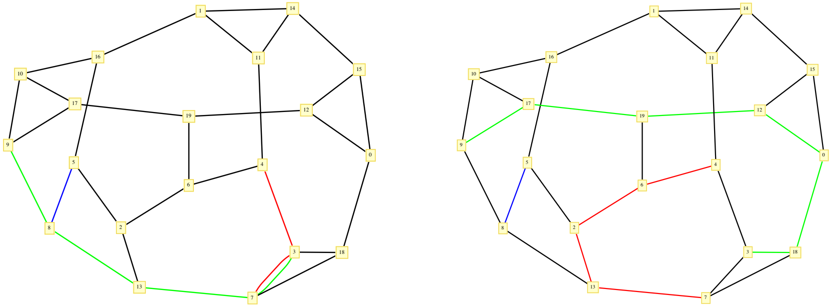
- One λ per communication
- Two communications sharing one link must be assigned two different λ's
- ... and the shorter the route the faster the communication

State of the art.

- **Integer Linear Programming** : expensive and unfeasible for larger systems
- **Greedy**: fast but poor performing
- **More structured algorithms:** Ant Colony Optimizations, Stimulated Annealing, differential evolution: not so fast, too many parameters ...

Node disjoint path (NDP)

Decision problem: is there a NDP configuration that accommodates all the communications?



Shortest node-disjoint paths on random graphs, C De Bacco, S Franz, D Saad and C H Yeung. J. Stat. Mech. (2014) P07009

Cavity Max-Sum equations.

$\bar{I}_{ij} := (I_{ij}^1, \ldots, I_{ij}^M)$ flow along edge (ij), $M=$ number of communications

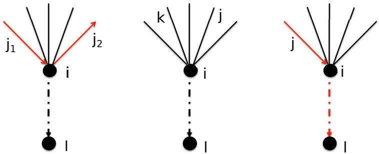
$$E_{ij}(\bar{I}_{ij}) = \min_{\bar{I}_{ki}|constraint} \left\{ \sum_{k \in \partial i \setminus j} E_{ki}(\bar{I}_{ki}) \right\} + f(||\bar{I}_{ij}||) \quad (1)$$

$$I_{ij}^{\mu} = \begin{cases} +1 & \text{if } \mu \text{ passes through } (ij) \text{ from } i \text{ to } j \\ -1 & \text{if } \mu \text{ passes through } (ij) \text{ from } j \text{ to } i \\ 0 & \text{if } \mu \text{ does not pass through } (ij) \end{cases} \quad (2)$$

$$f(||\bar{I}||) = \begin{cases} \infty & \text{if } ||\bar{I}|| \geq 2 \\ 1 & \text{if } ||\bar{I}|| = 1 \\ 0 & \text{if } ||\bar{I}|| = 0 \end{cases} \quad (3)$$

Constraint is Kirchhoff law

Cavity Max-Sum equations.



If $|\bar{\Lambda}_i| = 0$ **then:**

$$E_{il}(\bar{I}_{il} = \bar{0}) = \min \left\{ \sum_{j \in \partial i \setminus l} E_{ji}(\bar{I}_{ji} = \bar{0}), \right. \tag{4}$$

$$\left. \min_{j_1, j_2 \in \partial i \setminus l; \mu \in M} \left[E_{j_1 i}(I^{\mu}_{j_1 i} = +1) + E_{j_2 i}(I^{\mu}_{j_2 i} = -1) + \sum_{k \in \partial i \setminus l, j_1, j_2} E_{ki}(\bar{I}_{ji} = \bar{0}) \right] \right\}$$

$$E_{il}(I^{\mu}_{il} = \pm 1) = \min_{j \in \partial i \setminus l} \left\{ E_{ji}(I^{\mu}_{ji} = \pm 1) + \sum_{k \in \partial i \setminus l, j} E_{ki}(\bar{I}_{ki} = \bar{0}) \right\} + 1 \tag{5}$$

If $\Lambda^{\mu}_i = \pm 1$ **then:**

$$E_{il}(\bar{I}_{il} = \bar{0}) = \min_{j \in \partial i \setminus l} \left\{ E_{ji}(I^{\mu}_{ji} = \mp 1) + \sum_{k \in \partial i \setminus l, j} E_{ki}(\bar{I}_{ki} = \bar{0}) \right\} \tag{6}$$

$$E_{ji}(I^{\nu}_{ji} = \pm 1) = +\infty \quad (\nu \neq \mu) \tag{7}$$

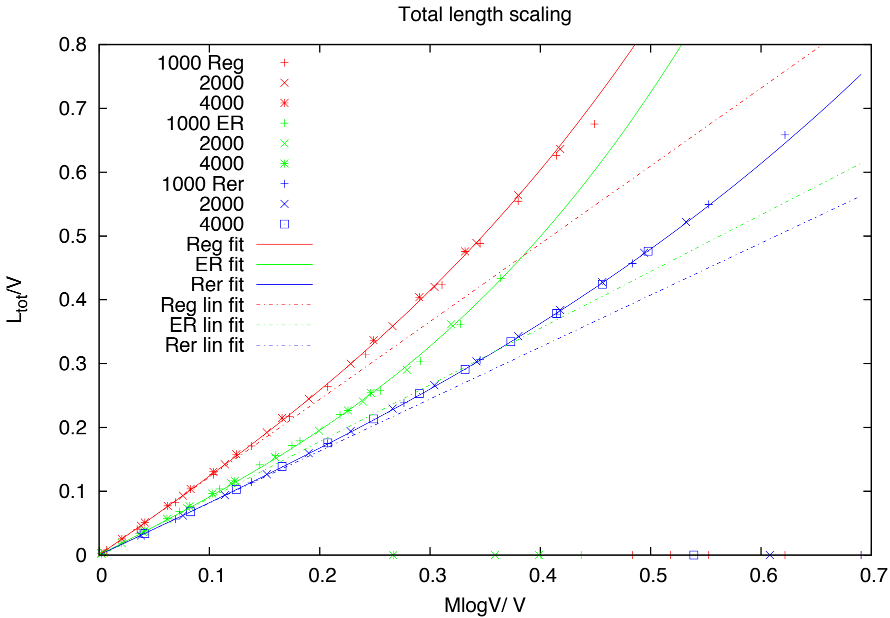
$$E_{ji}(I^{\mu}_{ji} = \mp 1) = +\infty \tag{8}$$

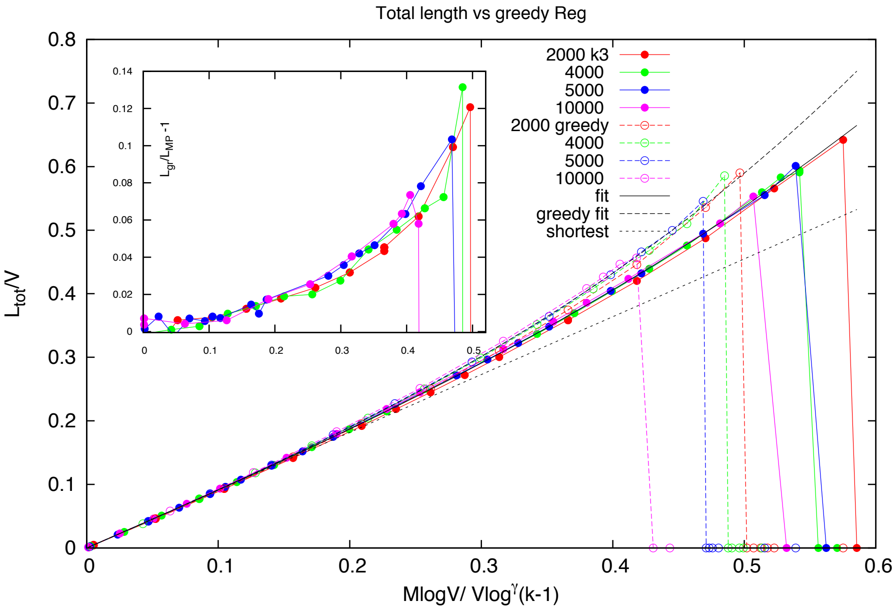
Solution after convergence

$$E_{ij}^{Link}(\bar{I}) := E_{ij}(\bar{I}) + E_{ji}(-\bar{I}) - ||\bar{I}|| \tag{10}$$

$$\bar{I}_{ij}^{*} := arg\ min_{\bar{I}} E_{ij}^{Link}(\bar{I}) \tag{11}$$

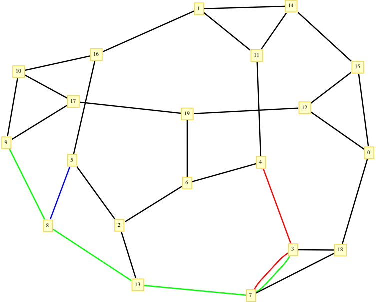
$$L_{tot} := \sum_{(ij)\in\mathcal{E}} ||\bar{I}_{ij}^{*}|| \tag{12}$$

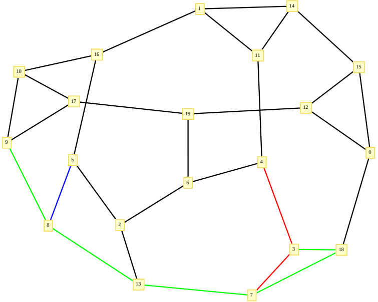

Total length scaling
1000 Reg
2000
4000
1000 ER
2000
4000
1000 Rer
2000
4000
Reg fit
ER fit
Rer fit
Reg lin fit
ER lin fit
Rer lin fit
L_{tot}/V
MlogV/ V
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8


Total length vs greedy Reg
L_{tot}/V
$M\log V/\ V\log^{\gamma}(k-1)$
2000 k3
4000
5000
10000
2000 greedy
4000
5000
10000
fit
greedy fit
shortest
L_{gr}/L_{MP} -1
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.14
0.12
0.1
0.08
0.06
0.04
0.02

Edge disjoint path (EDP)

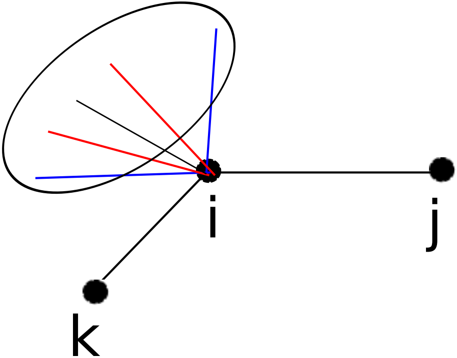
Optimization problem: what is the max number of communications accommodated?
(c.f. in the NDP the focus was on the decision problem)





Edge disjoint path (EDP)

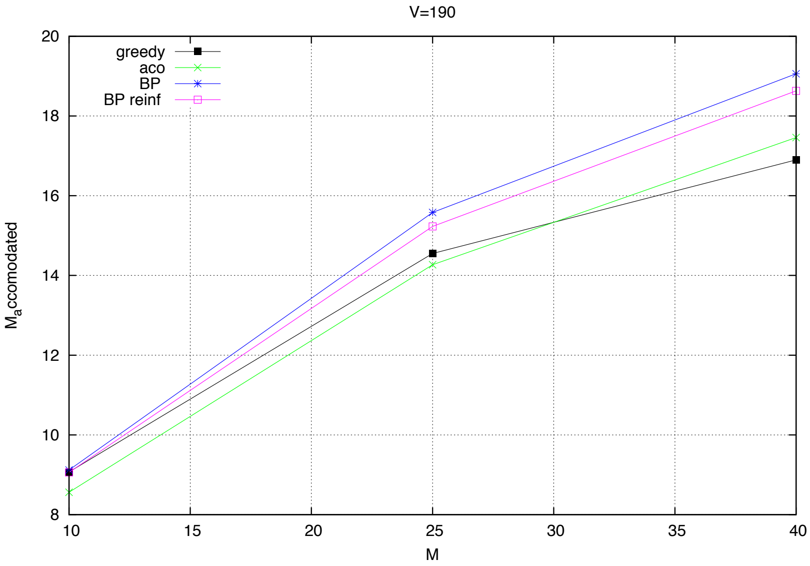
The number of possible configurations is exponential in k: considering edge (ij) and a given neighbor $k \in \partial i \setminus j$, all the remaining $k-2$ flows are left free...

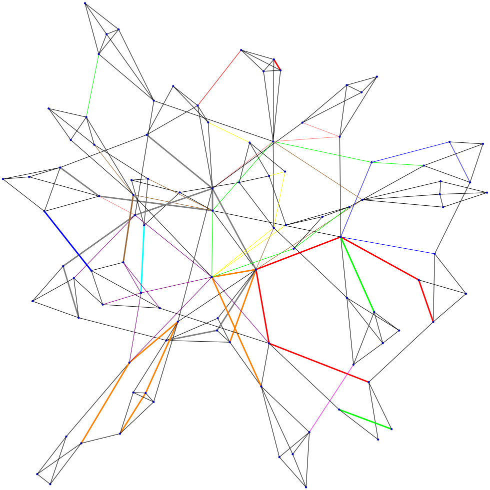


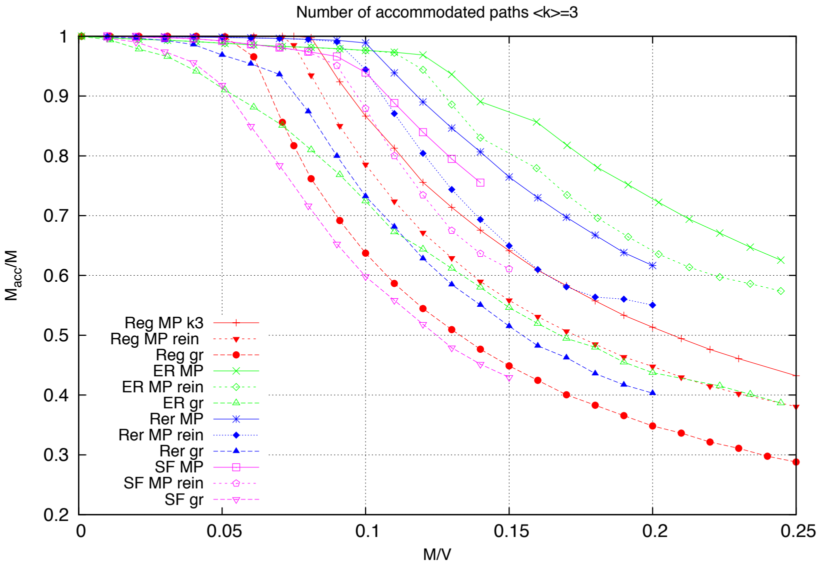
In collaboration with Polito, starting during the secondement and in phase of finalization: combinatorial algorithm that solves the problem with complexity

$$O(k^5 + Mk^2)$$

Only benchmark found: ACO (ant colony optimization) on random graphs generated through BRITE (open source routine for network generation developed at Boston University).

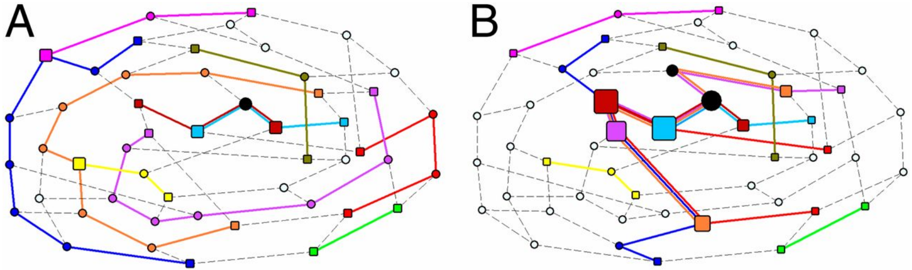

V=190
greedy
aco
BP
BP reinf
20
18
16
14
12
10
8
10
15
20
25
30
35
40
M
$M_a$ccomodated




Number of accommodated paths <k>=3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.05
0.1
0.15
0.2
0.25
M/V
M_{acc}/M
Reg MP k3
Reg MP rein
Reg gr
ER MP
ER MP rein
ER gr
Rer MP
Rer MP rein
Rer gr
SF MP
SF MP rein
SF gr

Congestion optimization

Final step: remove disjointness constraint.



From the physics of interacting polymers to optimizing routes on the London Underground, CH Yeung, D Saad and K Y M Wong PNAS 2013 110 (34)

Mean field approximation

Cavity equations:

$$E_{ij}(\bar{I}_{ij}) = \min_{\bar{I}_{ki}|\textcolor{red}{constraint}} \left\{ \sum_{k \in \partial i \setminus j} E_{ki}(\bar{I}_{ki}) \right\} + f(||\bar{I}_{ij}||) \tag{13}$$

Idea: suppose μ passes through (ij). Replace the neighborhood flow configuration $||\bar{I}_{ki}||$ with a mean field value $\lambda_{ki}^{\mu MF}$. This value is obtained by solving a self-consistent equation:

$$\lambda_{ki}^{\mu MF} = 1 + \sum_{\nu \neq \mu} \Theta(g(\lambda_{ki}^{\nu MF}, \{E_{k'i}^{\mu}(I)\})) \tag{14}$$

where g is a function of the other neighbors $k' \in \partial i \setminus k, j$ messages, it evaluates whether it is convenient or not for the other $\nu \neq \mu$ to pass along (ki) conditioned to the fact that μ is passing.

Conclusion

1. RWA in optical network: decision and optimization problems.
2. Node disjoint path: decision problem.
3. Edge disjoint path: optimization problem.
4. Congestion optimization: work in progress.