Anderson localization of light

and inference???

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PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

- Above a certain amount of disorder no transport is possible „Anderson localization"

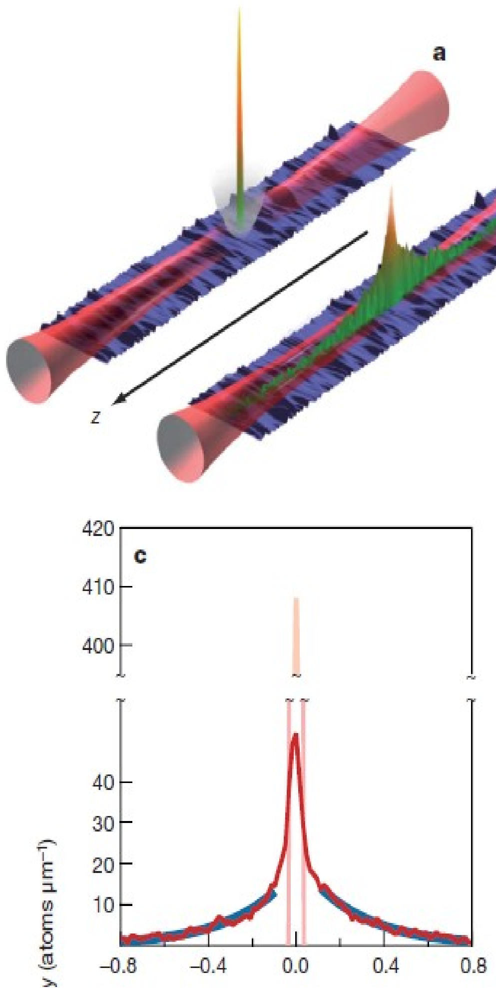
- The reason: localized states due to disorder

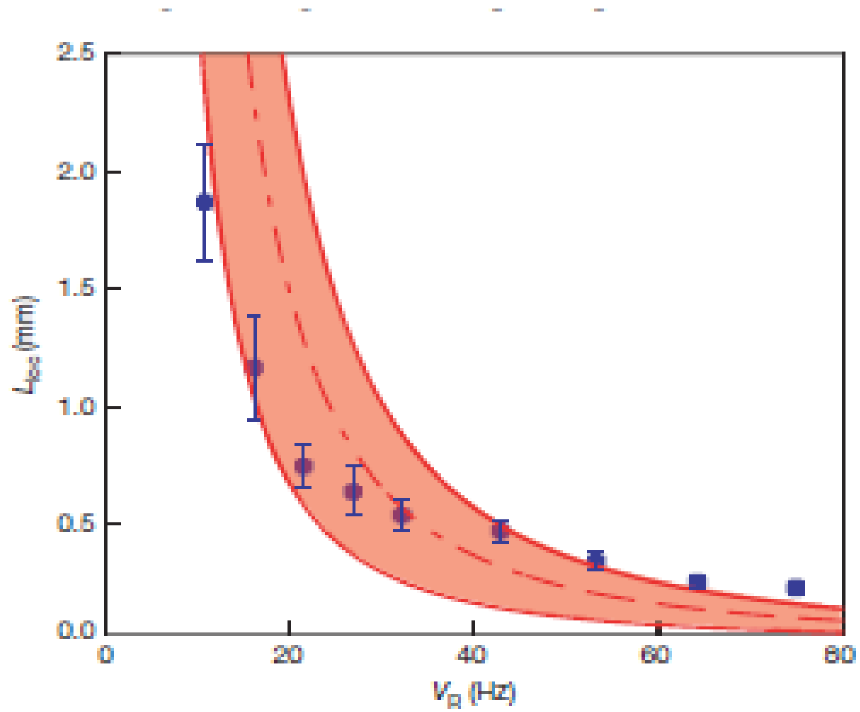
Literature

- Observation of Anderson localization in
  - Nonlinear Optics
    - Y. Lahini et al. PRL 100, 013806 (2008)
    - T. Schwartz, G. Bartal, S. Fishman, M. Segev, Nature 446, 52 (2007)
  - Bose-Einstein condensation
    - J. Billy et al. Nature 453, 891 (2008)
    - G. Roati et al. Nature 453, 895 (2008)
    - S. S. Kondov, Science 66, 334 (2011)
  - Linear disordered media (optics)
    - M. Storzer, P. Gross, C. M. Aegerter, G. Maret, PRL 96, 063904 (2006)
    - A. A. Chabanov, M. Stoytchev, A. Z. Genack, Nature 404, 850 (2000)
    - T. Sperling at al, Nature Photonics 7, 48 (2013)

1D Bosons (BEC)

- Billy et Nature 2008





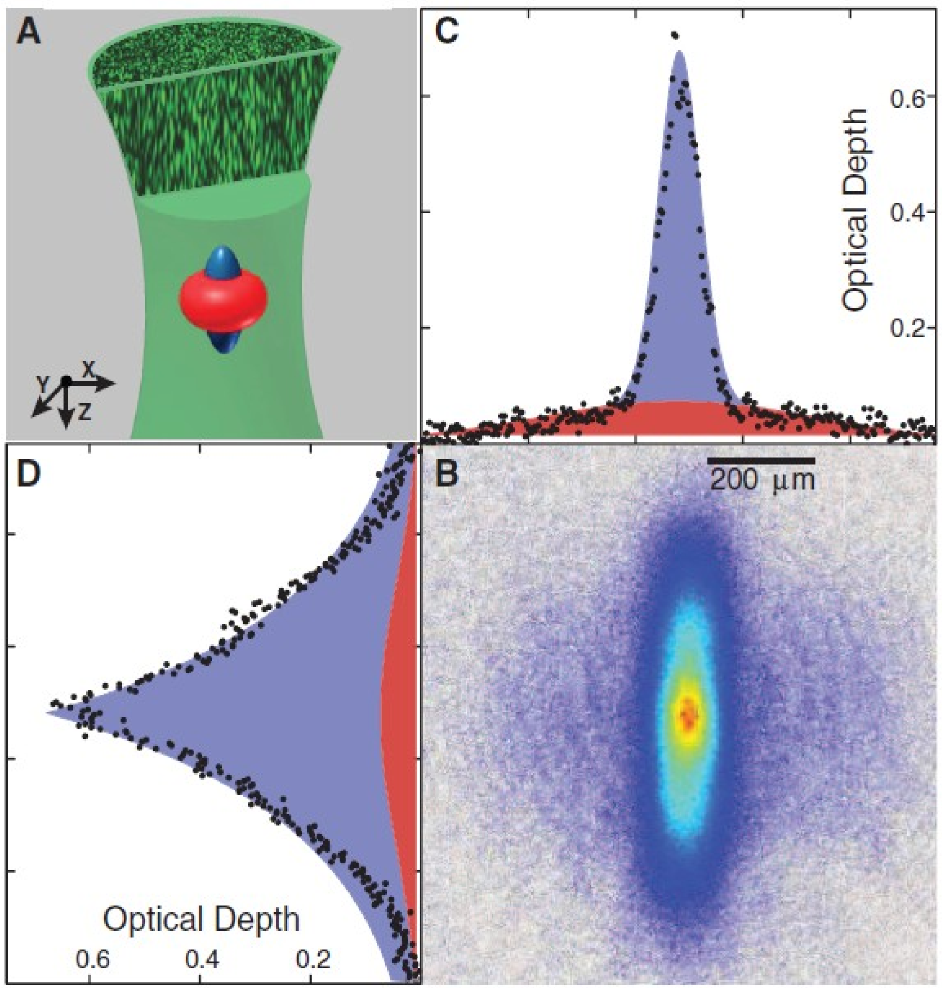
Localization length versus strenght of disorder

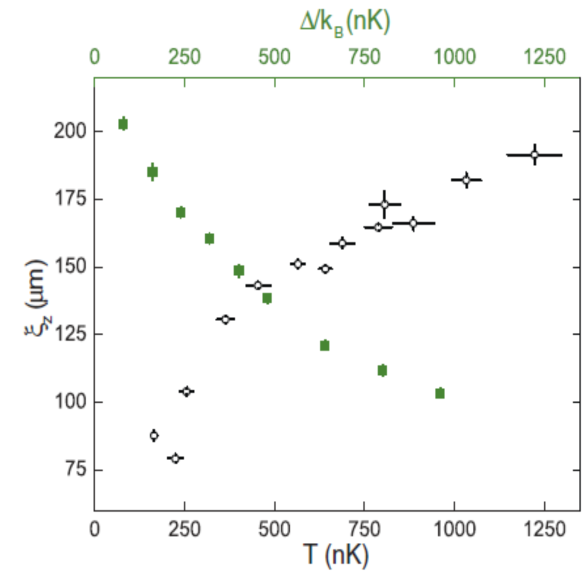
Also Roati et al Nature 2008

3D Fermions (BEC)

- Kondov et al. Science 2011

Fig. 1. (**A**) Ultracold gas expanding into an optical speckle field (green) and separating into localized (blue) and mobile (red) components. (**B**) The measured optical depth, proportional to the atomic density integrated through *y*, is shown in false color. The image depicts a 480-nK gas that has expanded for 20 ms through the disordered potential with $\Delta = k_B \times 240$ nK. All images shown in this manuscript are averaged over at least five experimental realizations. Slices are shown through the image along *x* (**C**) and *z* (**D**). The filled curves are fits to independent mobile (red) and localized (blue) components.





Localization length
Versus disorder

3D Photon

- Sperling et al.
  Nature Photonics 2013

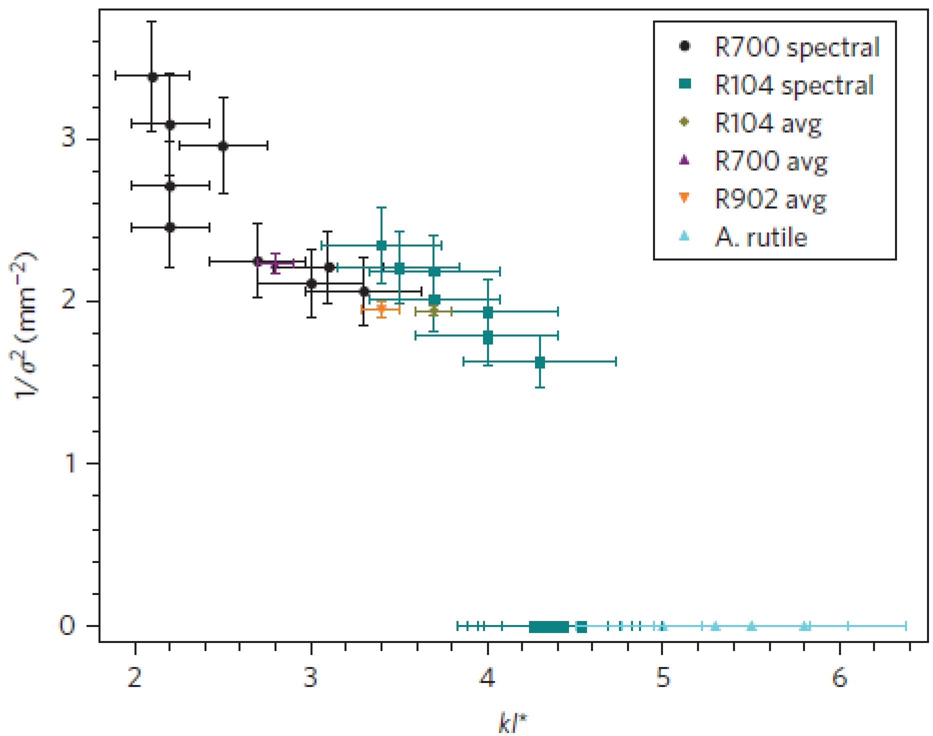


Figure 4 | Inverse of the mean-square width σ^2_∞ of the plateau versus kl^* for different samples. As can be seen, the width (corresponding to the localization length) diverges at $l^* \approx 4.5$, indicating the transition from a localized to a non-localized state. The increase of the localization length approaching the critical turbidity can also be used to estimate the critical exponent. All error bars correspond to systematic errors.

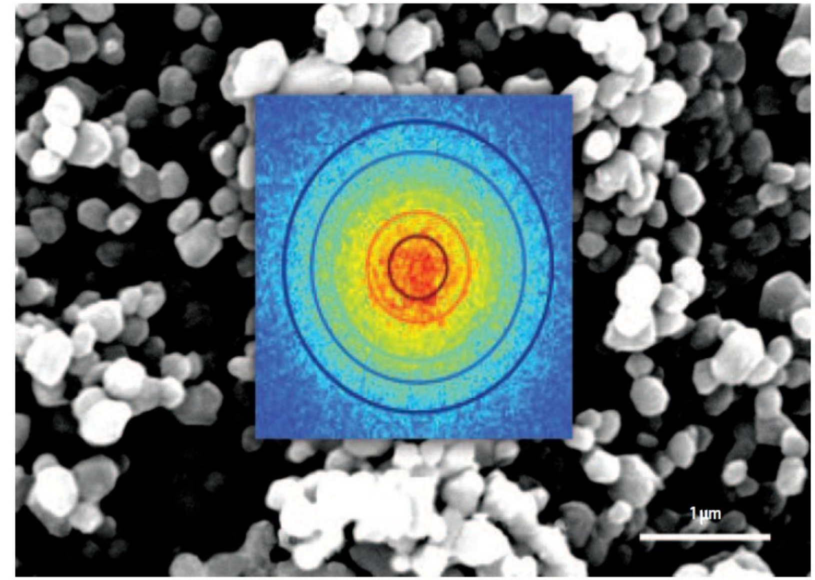
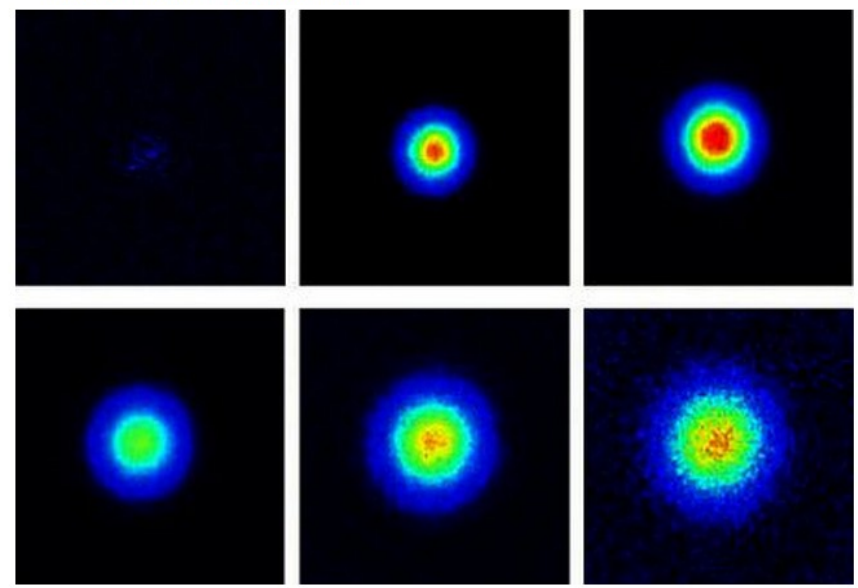


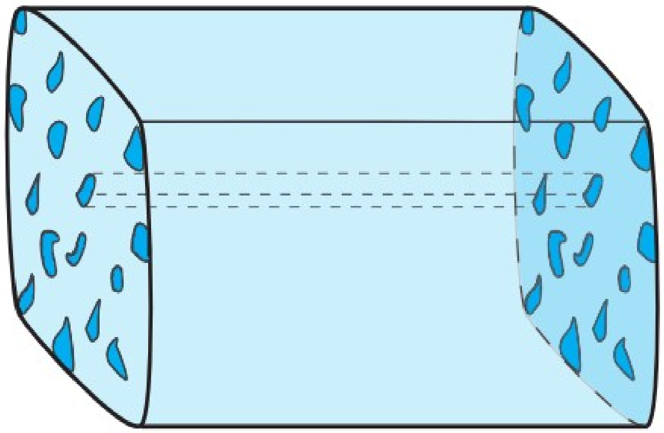
Figure 1 | Light at the onset of the Anderson localization superimposed over a scanning electron microscopy image of a disordered sample.



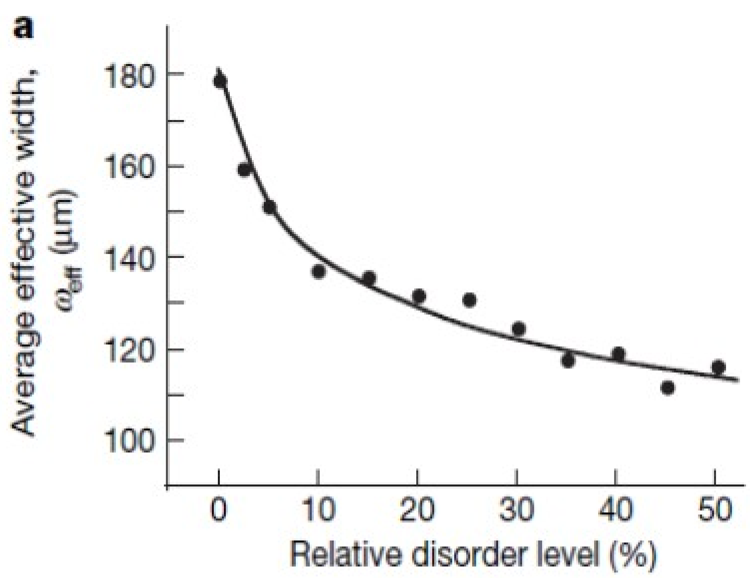
Diffusion of light in a disordered, cloudy medium at intervals of 1 ns. After about 4 ns, the light stops spreading any further. (*Courtesy of the University of Zurich*)

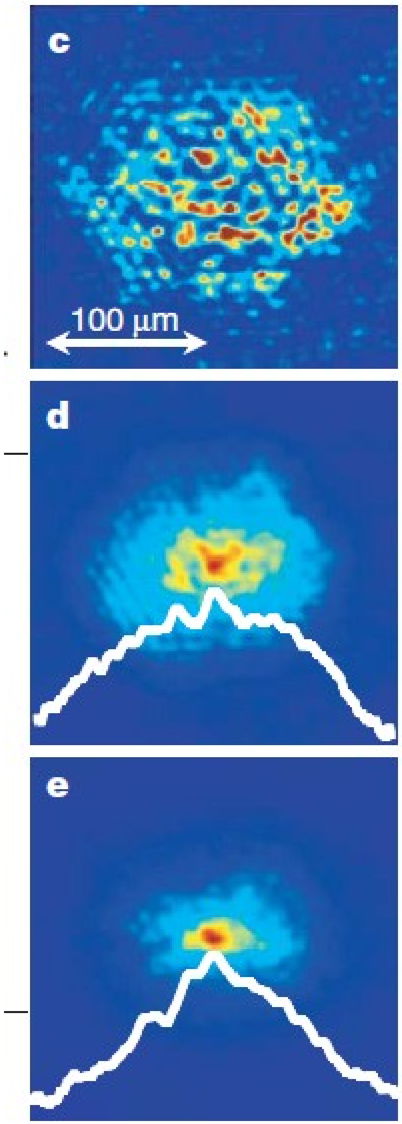
TRANSVERSE Anderson Loc

T. Schwartz, G. Bartal, S. Fishman, M. Segev, Nature 446, 52 (2007)



INDEX CONTRAST 0.0001
PROPAGATION 1cm





The simplest model

The model

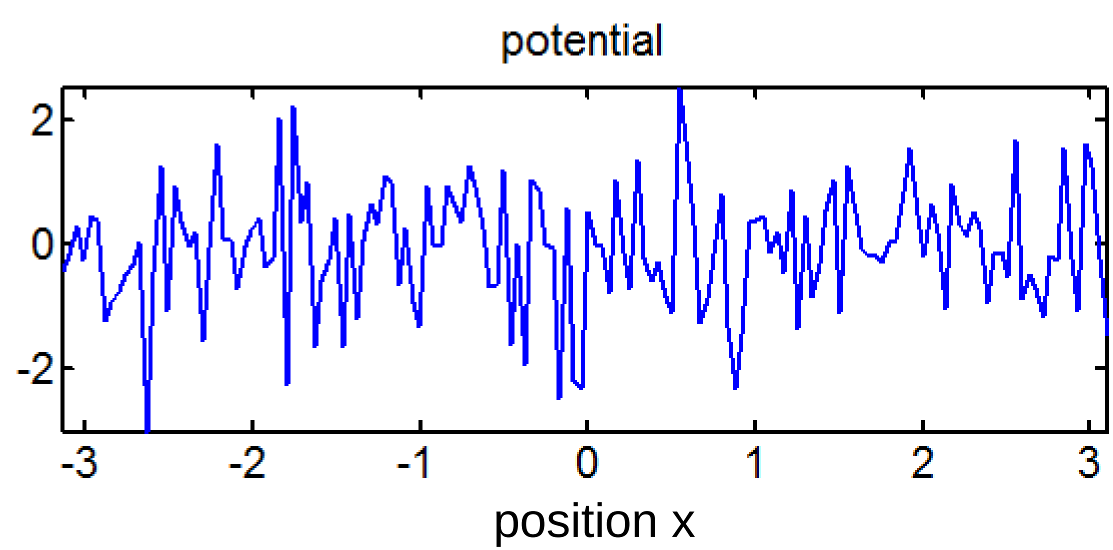
- One-dimensional NLS with a random potential

$$i\psi_t = -\psi_{xx} + V(x)\psi - \chi|\psi|^2\psi$$

$\chi = 0$	Linear
$\chi = +1$	Focusing
$\chi = -1$	Defocusing

$$P = \int |\psi|^2 dx$$

$$\langle V(x)V(x')\rangle = V_0^2\delta(x - x')$$



CC, PRA 86, 061801R (2012)

Nonlinear Anderson localization

- Bound state equation

$$\psi = \varphi \exp(-iEt)$$

$$-\varphi_{xx} + V(x)\varphi - \chi\varphi^3 = E\varphi,$$

- This is solved numerically by a pseudo-spectral Newton-Raphson algorithm

The simplest Anderson localization

$$-\varphi_{xx} + V(x)\varphi = \mathcal{L}\varphi = E\varphi,$$

$$\chi = 0$$

- One dimensional LINEAR Schroedinger equation with random potential
  - Specific case:
    - a Gaussianly distributed random potential
  - Known issues:
    - Existence of exponentially localized states (negative eigenvalues)
    - Distribution of eigenvalues
    - Localization length

Linearly localized states

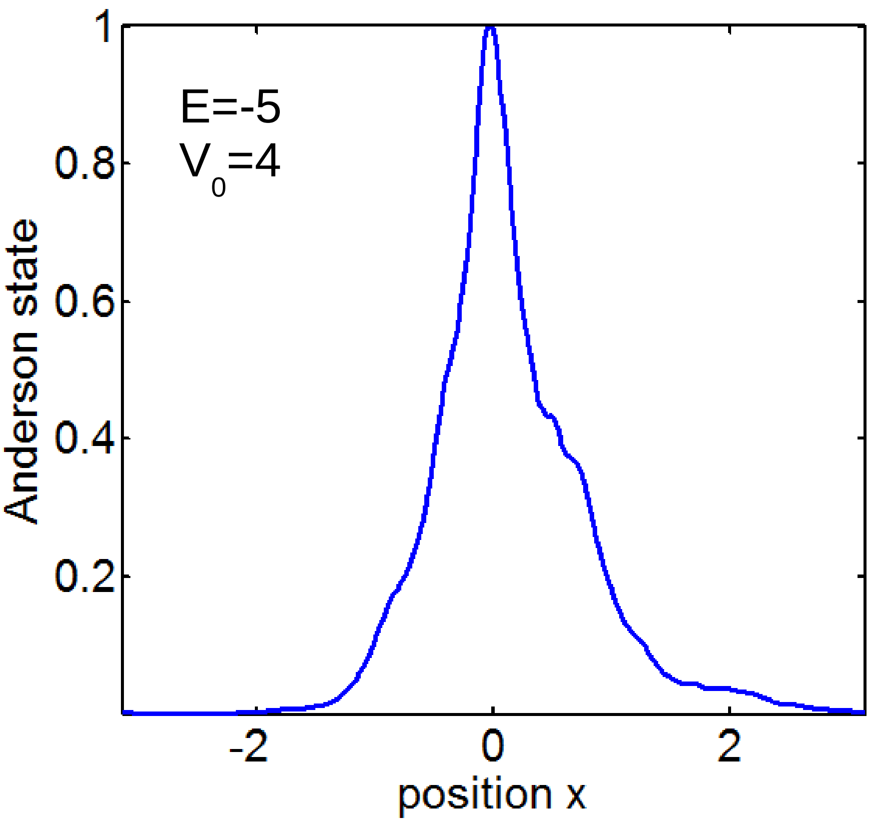
- Gaussian potential
- Negative eigenvalues
- Decays as $\exp(-\sqrt{-E}|x|)$
- **Link between**

  localization length

  and

  eigenvalue

$$-\varphi_{xx} + V(x)\varphi = \mathcal{L}\varphi = E\varphi,$$



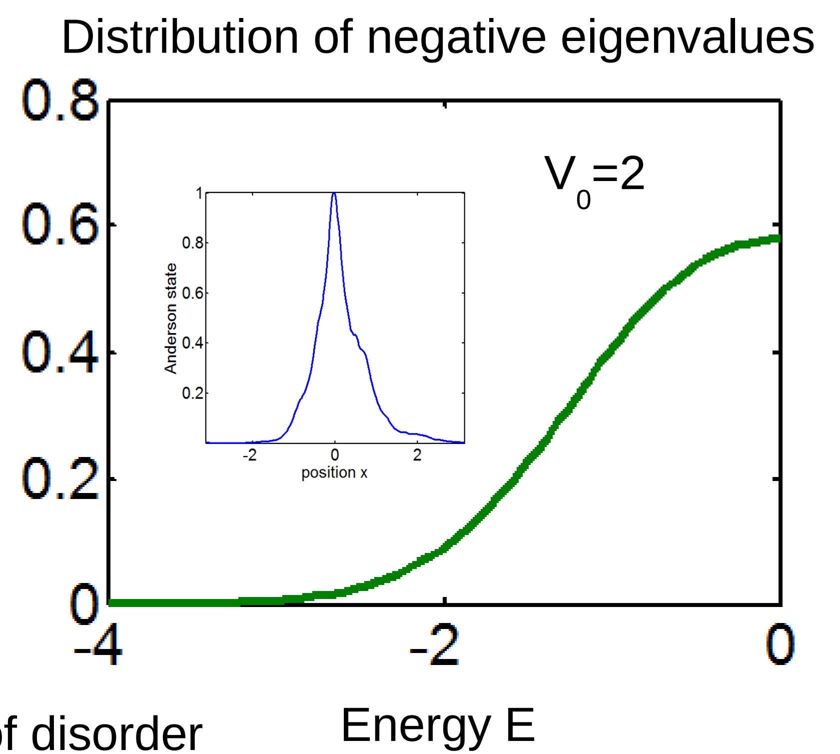
The statistical distribution of eigenvalues

- There is a tail of negative energies corresponding to

  exponentially highly localized states

$$\langle V(x)V(x')\rangle = V_0^2\delta(x-x')$$

$$\overline{E}_L \cong -V_0^{4/3}/3$$



The localization length decreases as the Inverse square root of the |energy|, hence the localization length decreases with the amount of disorder (as observed experimentally)

Localization length l

- It is calculated by the inverse participation ratio

$$l = \frac{|\int \varphi^2 dx|^2}{\int \varphi^4 dx} = \frac{P^2}{\int \varphi^4 dx}$$

$$P = \int |\psi|^2 dx$$

- For an exponentially localized state

$$\varphi_e = \frac{e^{-2|x|/l}}{\sqrt{l/2}}$$

Link between localization length and eigenvalue in the LINEAR case

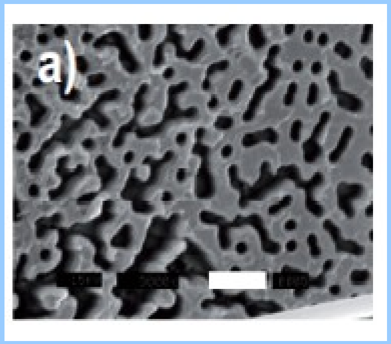
- The localization length scales as inverse squares root of the eigenvalue

$$l = \frac{3}{\sqrt{-E}}$$

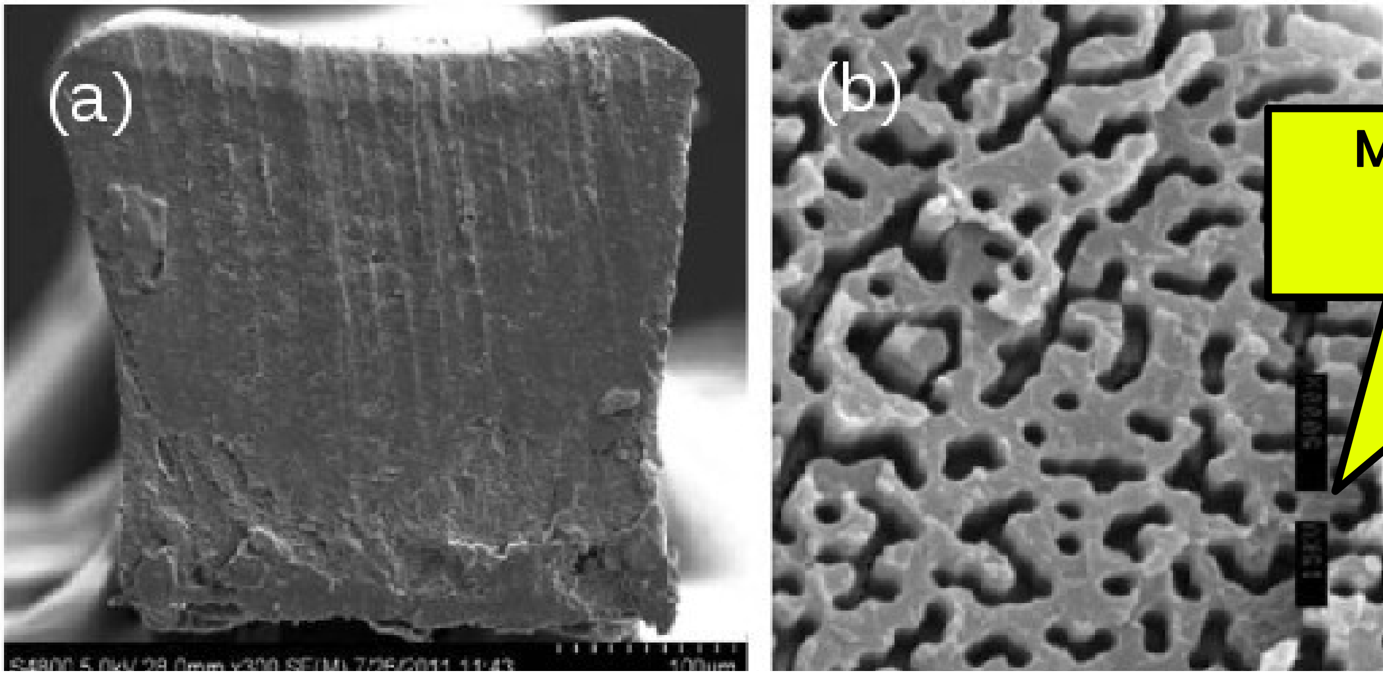
- The lower the negative energy,

  the more localized

Transverse localization in 2D fibers



Our experiments on transverse localization in two dimensional fibers

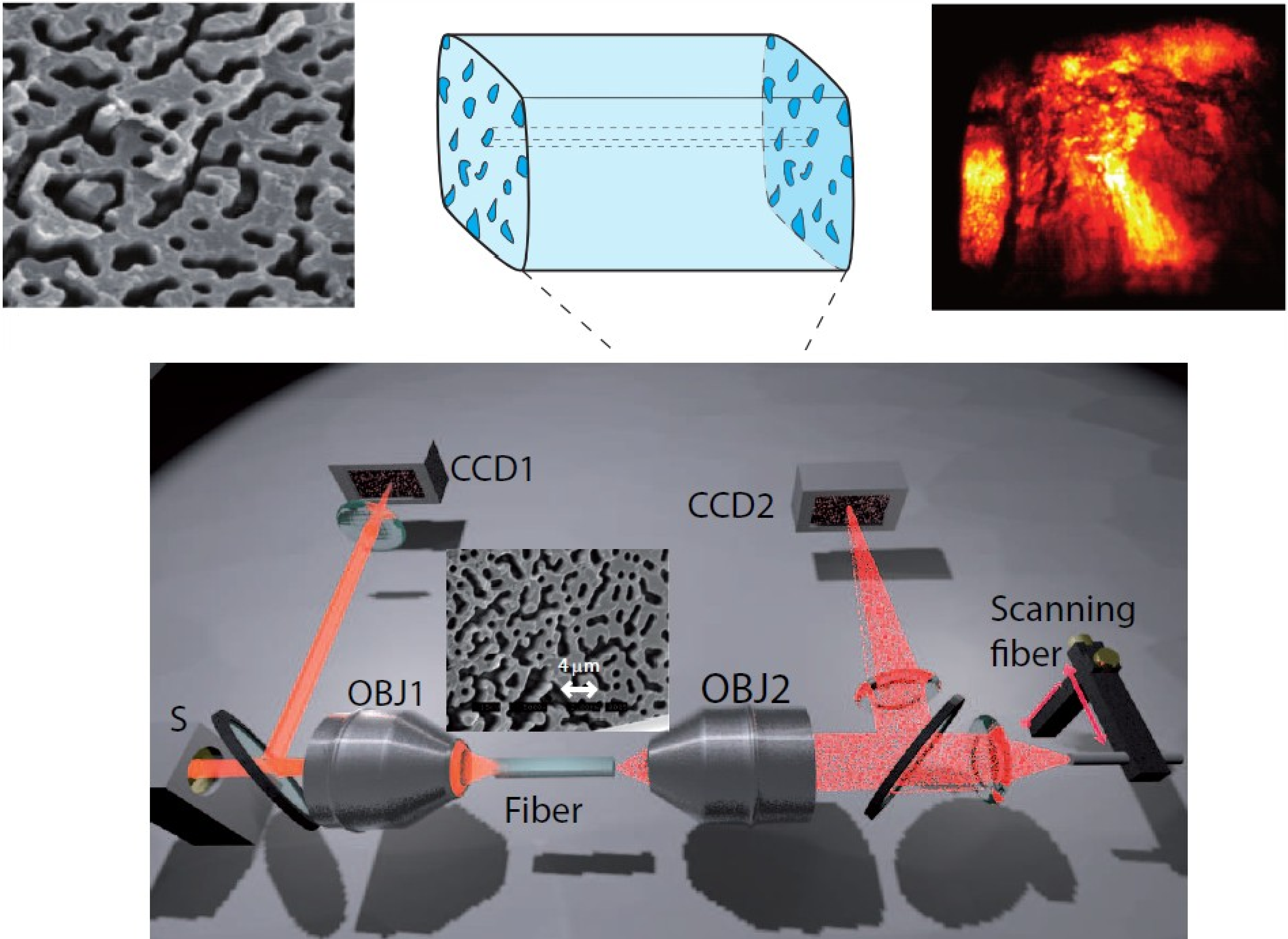


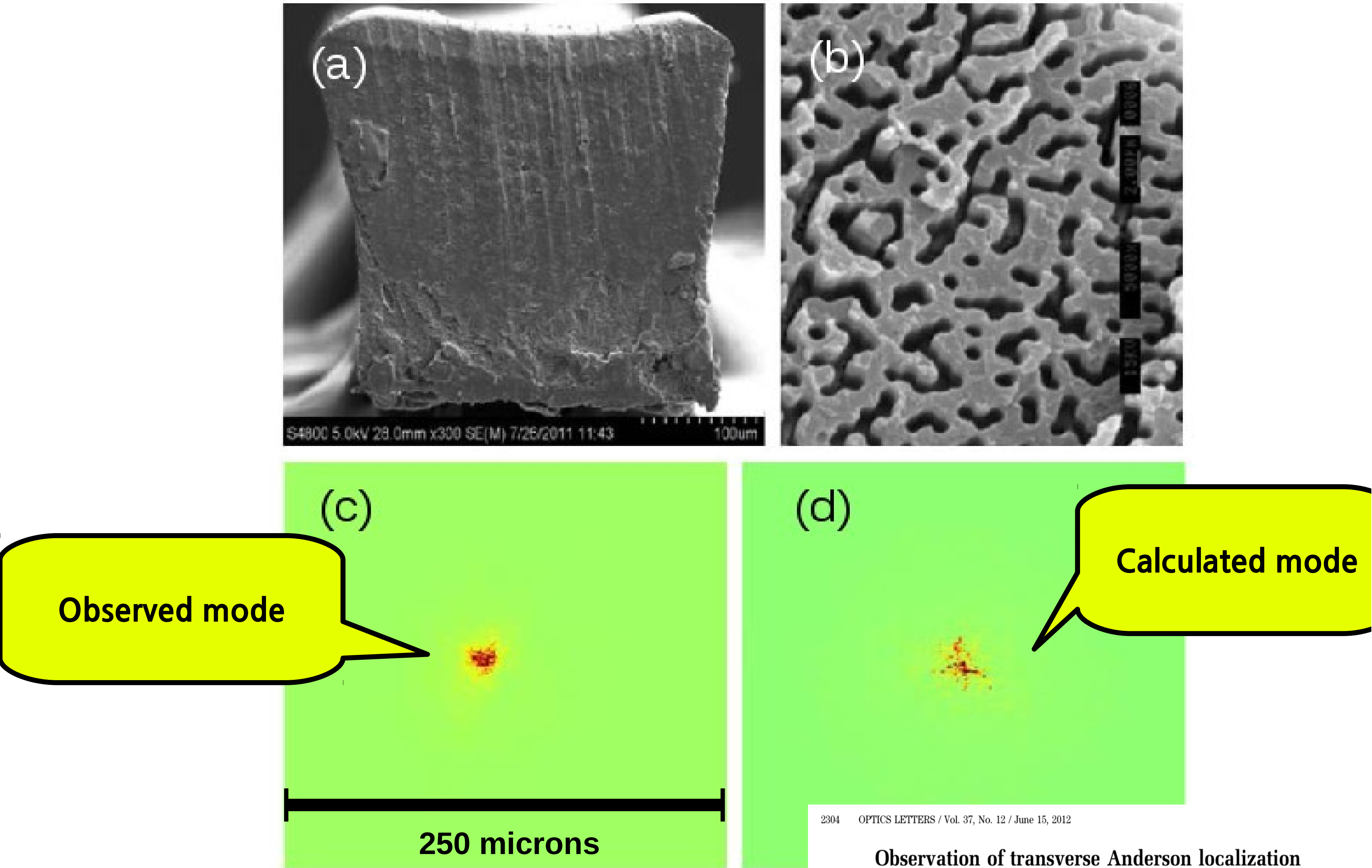
40000 pieces of PMMA and 40000 pieces of PS randomly mixed and fused together
n(PS)=1.59
n(PMMA)=1.49

2304 OPTICS LETTERS / Vol. 37, No. 12 / June 15, 2012

Observation of transverse Anderson localization in an optical fiber

Salman Karbasi,[1] Craig R. Mirr,[1] Parisa Gandomkar Yarandi,[1] Ryan J. Frazier,[1] Karl W. Koch,[2] and Arash Mafi[1,*]


CCD1
CCD2
Scanning
fiber
4 µm
OBJ1
OBJ2
S
Fiber



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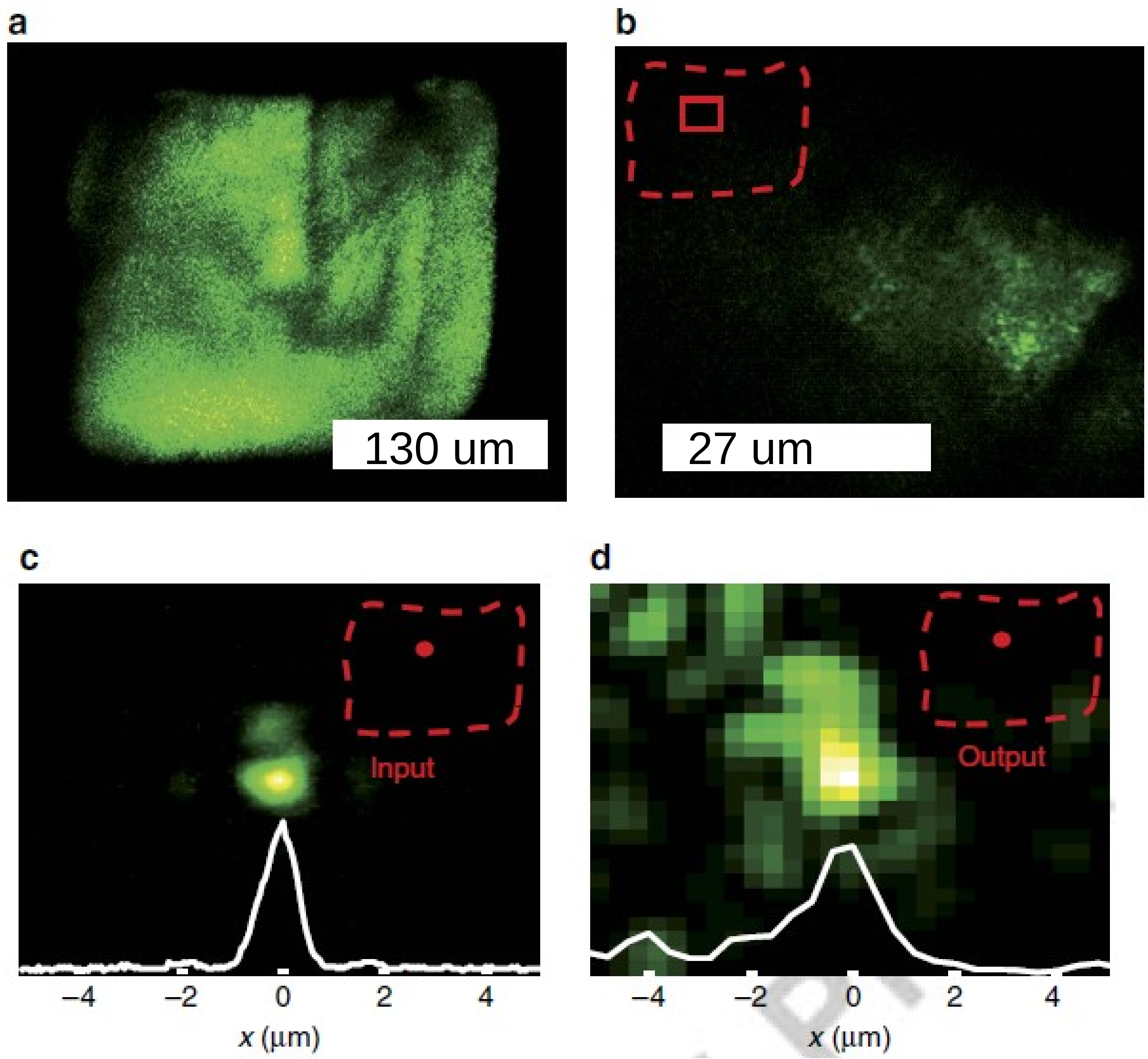
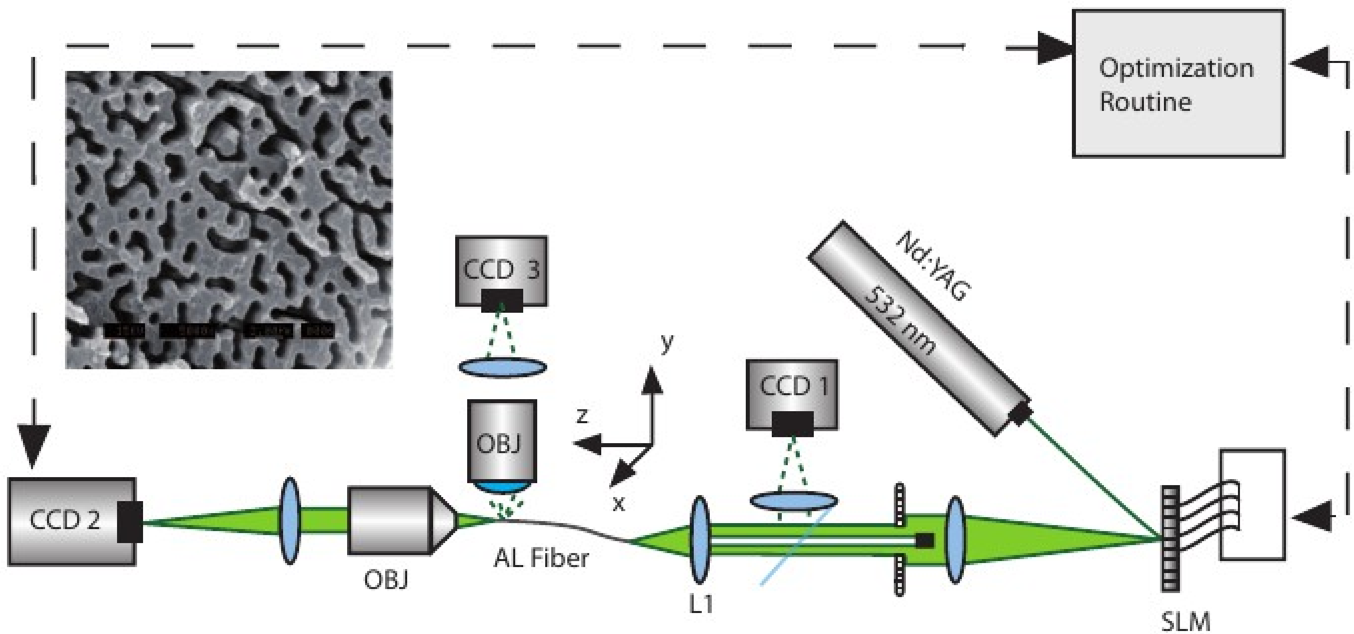
Absence of diffusion

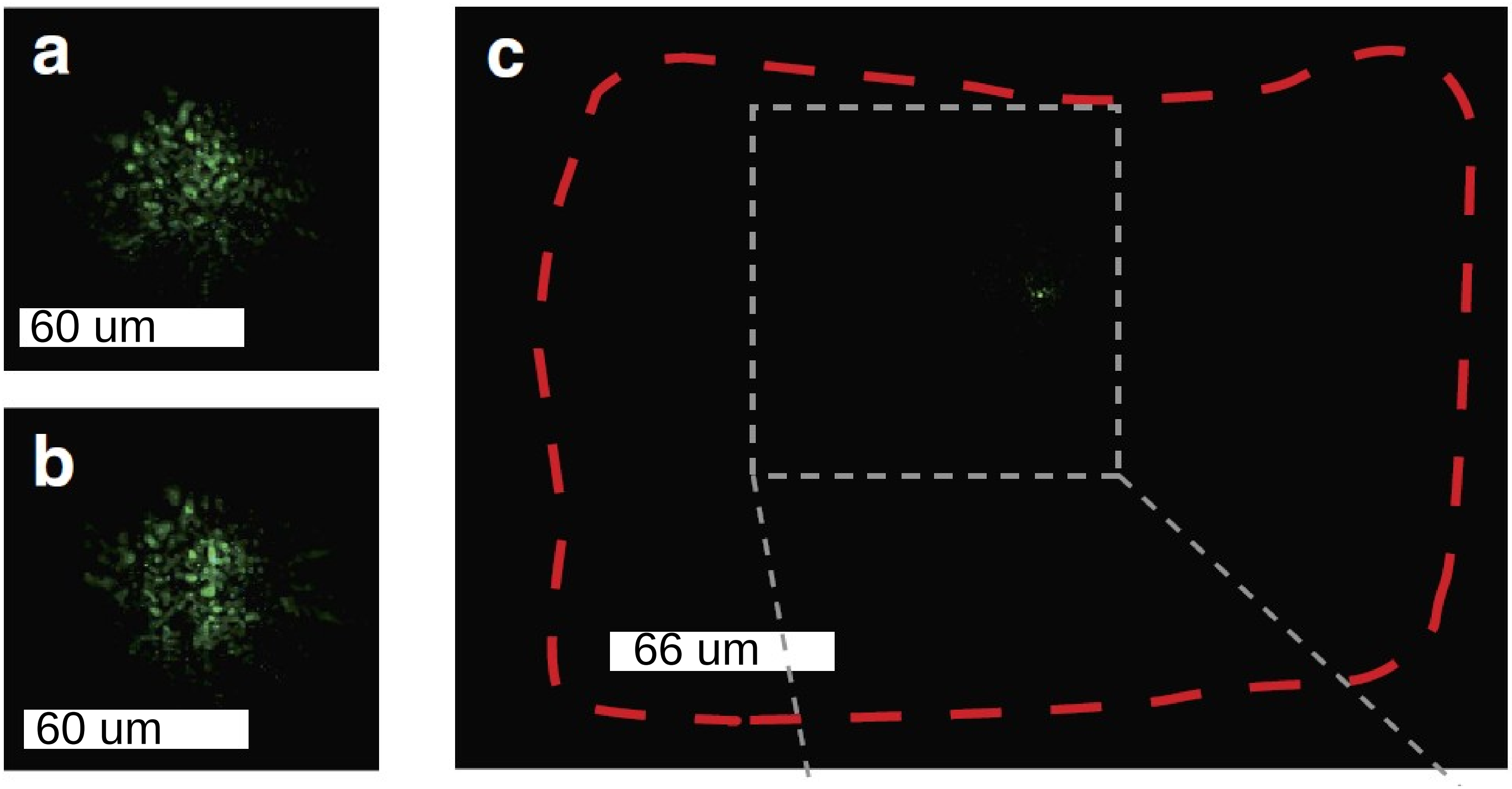


Optimization of light focusing through disordered media

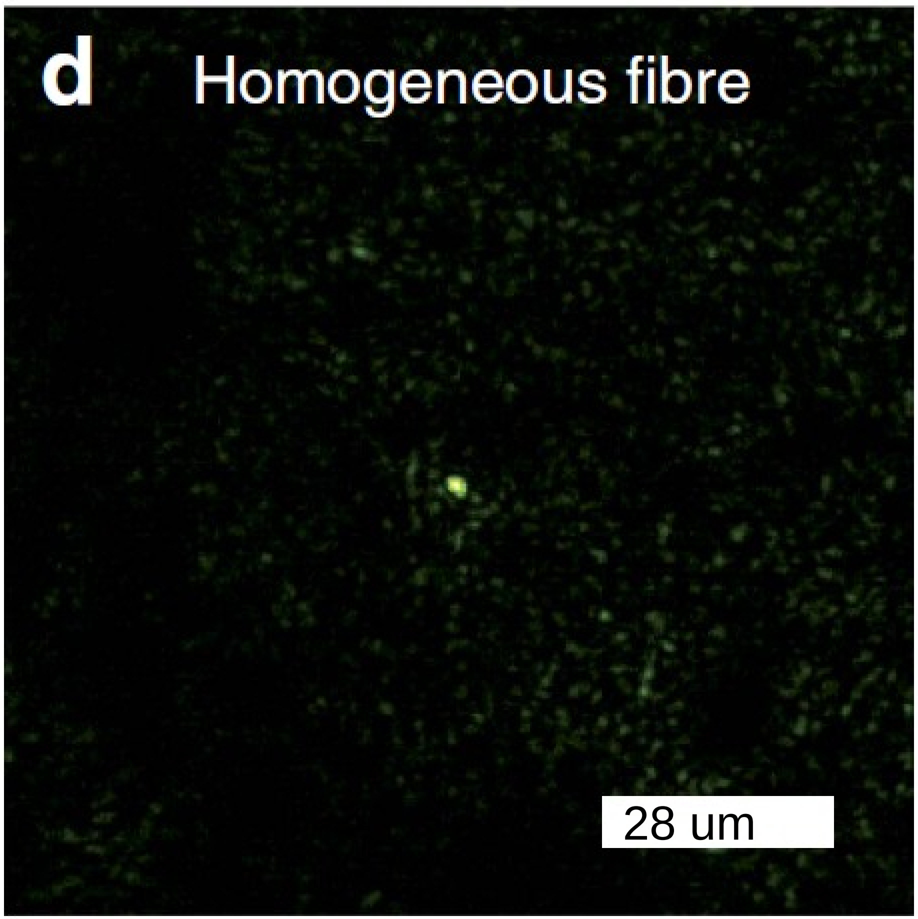
Focusing in the Anderson regime

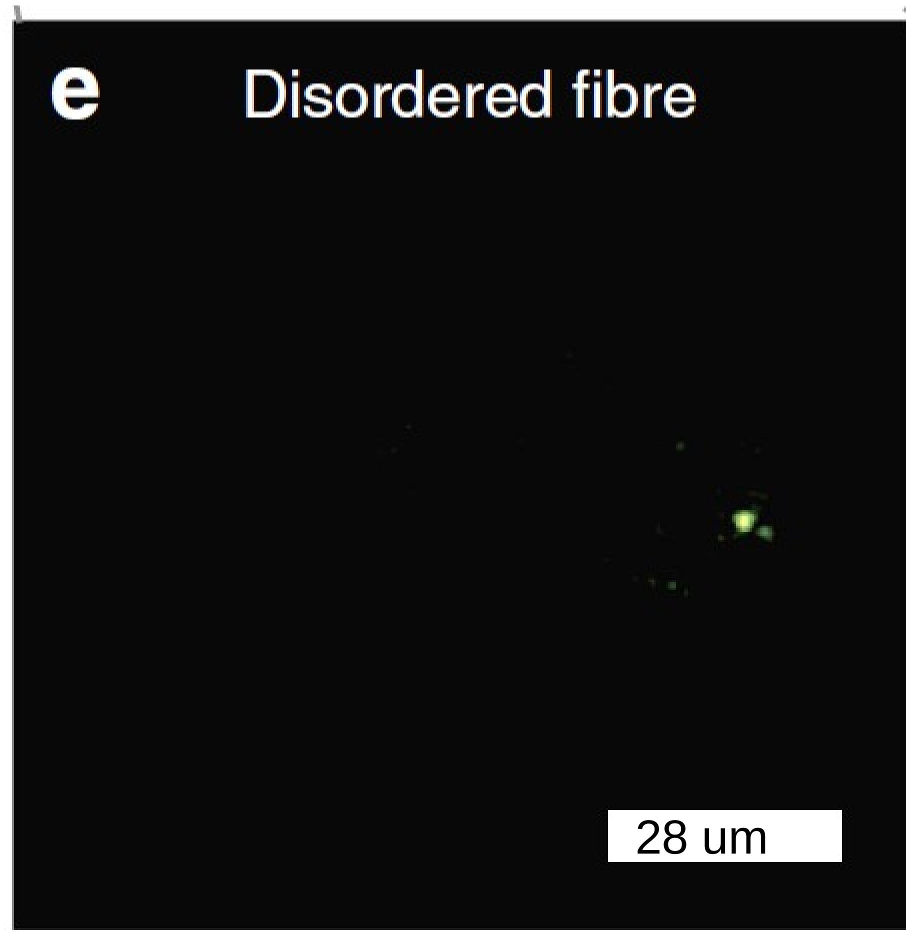


Adaptive focusing



Comparison





Further comparison

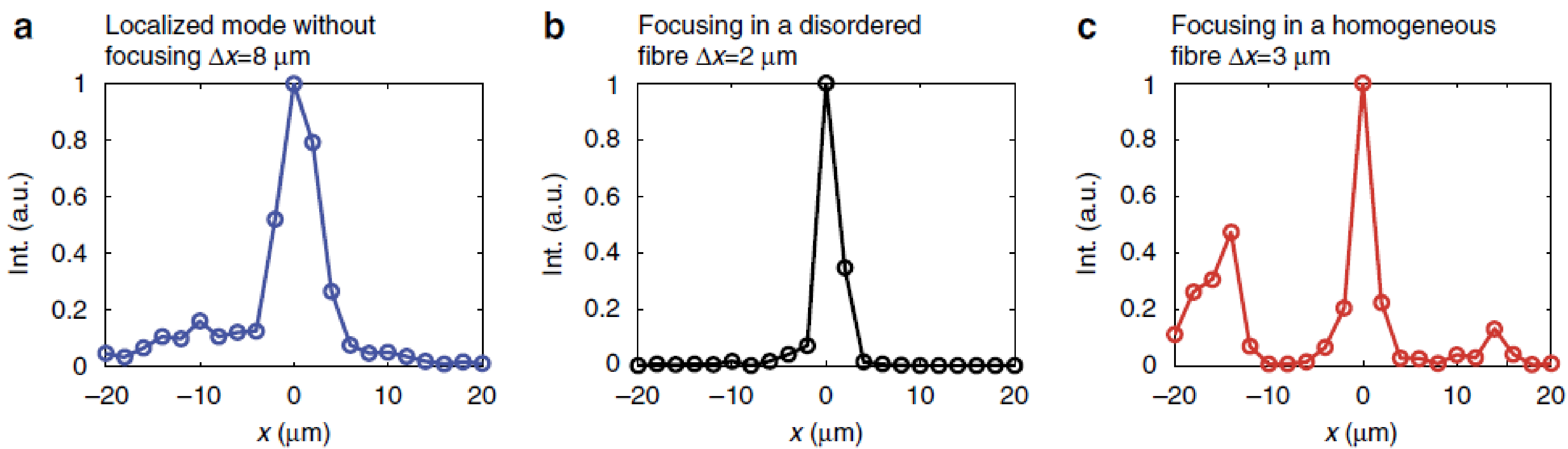


Figure 4 | Intensity profile of the focus. Profile of a localized mode (**a**) without optimization and (**b**) after the optimization procedure. (**c**) A focused mode in a homogeneous fibre; note the background.

Is this related to inference?

Measures from the top