

Inference of dynamical Gene Regulatory Networks and comparative genome analysis

Silvia Grigolon

Supervisors: Prof. *Silvio Franz* & Prof. *Olivier Martin*

LPTMS - Laboratoire de Physique Théorique et Modèles Statistique

—**Netadis Kick - Off Meeting**—
Turin, February 3rd – 6th, 2013

- 1 Where I come from...
 - My Origins
 - Past Scientific Background & Academic Interests
 - Brief Excursion into my Master's Thesis

- 2 What I do...
 - Current Scientific Background
 - Current work

- 3 What I am going to do...

My Origins



Past Scientific Background & Academic Interests

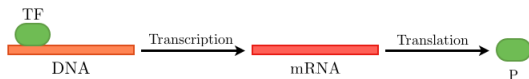
- ✓ November 2009, Laurea Triennale in Physics (BS) at *Sapienza, Università di Roma*;
- ✓ March 2012, Laurea Magistrale in Physics (MS) at *Sapienza, Università di Roma*:
 - specialization in Theoretical Physics – Statistical Mechanics;
 - Master's Degree Thesis on:

MicroRNA – based Networks and Circuits: a Noise Buffering Analysis,
under the supervision of Prof. *Enzo Marinari* and *Francesca Di Patti*, Ph.D.

I am interested in:

- * Systems Biology;
- * Statistical Mechanics (Classical and Quantum) and Disordered Systems;
- * Computational Physics;
- * Mathematical Physics and Dynamical Systems (especially what concerns chaotic systems and turbulence).

The role of Noise in Regulatory Processes

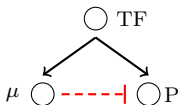


- **GRN** → set of genes (nodes of the network) interacting with each other through their produced proteins.
- **What is microRNA?** Small RNA molecule which inhibits genic expression.
- **What did we analyze?** **3 genetic circuits** involving microRNA to understand if this inhibitor behaves as a genetic fine tuner.

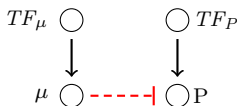
TF



FFL



Open



The degree of noise buffering depends on the **structure of the circuit**

- How did we study the problem?

Theoretically

van Kampen's Linear Noise Expansion

$$n_i \equiv \mathcal{N}\phi_i + \mathcal{N}^{\frac{1}{2}}\xi_i,$$

where:

- * n_i is the number of molecules of the i -species;
- * ϕ_i is the density of molecules in absence of fluctuations;
- * ξ_i is a random noise;
- * \mathcal{N} is the size of the system.

This permits to avoid all the three points correlations resulting from the second degree reaction involving microRNA.

Numerically

Gillespie's Algorithm

a *self-consistent algorithm on the time variable*:

it generates by itself the temporal step given as input all the reaction rates.

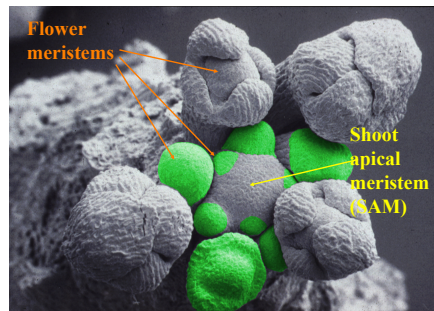
In the end, we found:

- a way to develop a **good theoretical framework** in agreement with numerical results;
- microRNA actually reduces protein fluctuations acting as a **fine tuner**.

S.Grigolon, F. Di Patti, A. De Martino, E. Marinari, *to be published*.

What is the genetic program for Floral Morphogenesis?

Arabidopsis Thaliana → Model Organism in plant biology and genetics



The overall conservation of flower structure suggests the existence and the persistence through evolution of **robust GRN modules** controlling the basic features of flower development.

Flower Development: the ABC model (Cohen & Meyerowitz, 1991)

4 Main and Highly Specified Structures in the Flower Meristem also known as **Floral Organs**:

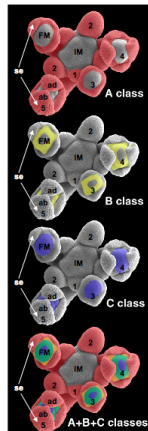
- Sepals;
- Petals;
- Stamens;
- Carpels.

ABC Model:

The genes involved in flower development are divided into **3 classes** named **A**, **B**, **C** that give rise to organs through these **FOS-GRN**

Interactions:

$$\begin{aligned}
 A &\rightarrow \text{Sepals,} \\
 A + B &\rightarrow \text{Petals,} \\
 B + C &\rightarrow \text{Stamens,} \\
 C &\rightarrow \text{Carpels.}
 \end{aligned}$$



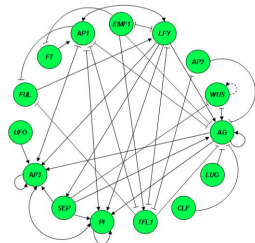
A first approach: a Boolean GRN (1)

E. R. Álvarez-Buylla et al., **PLoS One**, 3, 11, 2008

GRN extending the ABC model. It is aimed to obtain the gene expression profile of the steady states of the network.

$\underline{x}(t) \equiv$ state of the GRN, \mathcal{N} – components vector,

where $\mathcal{N}(= 15)$ is the number of genes in the GRN.



Stochastic Time-Evolution:

$$\forall n = \{1, \dots, \mathcal{N}\}, \frac{dx_n(t)}{dt} = \begin{cases} \alpha \cdot [F_n(\hat{x}_{n_1}, \dots, \hat{x}_{n_k}) - x_n(t)], & \eta \\ \alpha \cdot [1 - F_n(\hat{x}_{n_1}, \dots, \hat{x}_{n_k}) - x_n(t)], & 1 - \eta \end{cases}$$

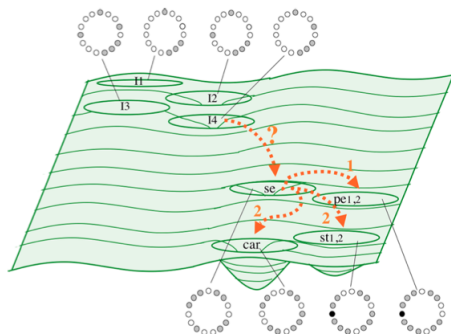
where:

- $F_n(\hat{x}_{n_1}, \dots, \hat{x}_{n_k})$ is a boolean variable-function guessed from microarray experiments;
- \hat{x}_n is a boolean variable connected to x_n (continuous variable) through the relation $\hat{x}_n = H(x_n - \theta_n)$;
- α is a constant connected to the relaxation time of the gene expression profile, τ ;
- η is a probability, defined to introduce the stochasticity in the model.

A first approach: a Boolean GRN (2)

This system of 15 differential equations has **10 attractors**:

- 6 corresponding to the gene expression profiles of sepals, petals (1,2), stamens(1,2) and carpels;
- 4 corresponding to the gene expression profiles of inflorescence configurations.



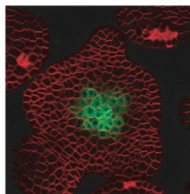
They obtained in particular the **probability transitions** among these attractors, caused by the presence of the noise η .

Why another model?

The previous model does not treat space and so there is no *spatio-temporal dynamics* → unable to explain the highly regular phyllotactic patterns?

Very recent biological studies:

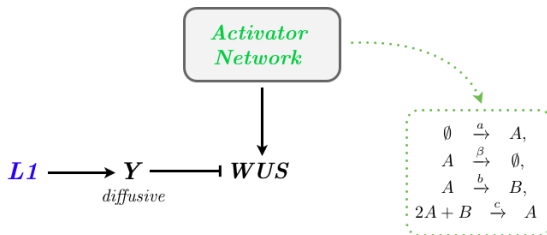
* F. Besnard, Ph.D. Thesis under the supervision of Prof. T. Vernoux, École Normale Supérieure de Lyon, 2011



- in floral morphogenesis a key compound regulating developmental processes: the **hormone auxin**, trigger of the initiation of floral primordia, depleted around them once they have been formed (*local inhibitory fields*);
- (*synergetic local inhibitory fields*) between Auxin and other genetic expression products (e.g., *WUS*);
- the time between two consecutive organ initiations is **NOT FIXED**;
- the observed patterns are **ROBUST** under environmental noise;
- co-initiations of organs could not be caused by a noise on the size of the involved fields but it depends on the noise of activations' thresholds.

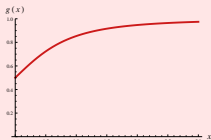
A second approach: a reaction–diffusion system

H. Jönsson et al., *Bioinformatics*, 21:1, i232, 2005



Activation Function:

$$g(x) = \frac{1}{2} \left(1 + \frac{x}{\sqrt{1+x^2}} \right)$$



L1 Layer:

$$L_1(x, y) = \begin{cases} 1, & (x, y) \in \partial\mathcal{L} \\ 0, & \text{otherwise} \end{cases}$$

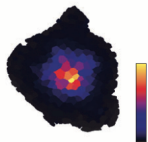
A and **B** → Brusselator molecules!

Boundary Conditions:

$$\nabla \cdot C_i = 0 \text{ on } \partial\mathcal{L}$$

A second approach: a reaction–diffusion system (2)

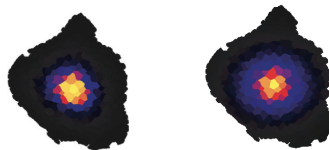
Experimental Results



WUSCHEL relative concentration

Numerical Results

5th Order Runge–Kutta Method with Adaptive Step Size



WUSCHEL concentration

A concentration



L1 Layer

Main Problem:

The simulated system is not robust to small parameter changes!

A simple diffusive model with thresholds

Main Assumption: Floral Morphogenesis is driven by **Auxin (A)** and **Wuschel (W)**.

Basic Features:

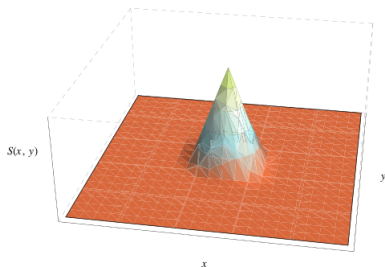
- * the phenomenon takes place in a 2D square lattice \mathcal{L} where the origin is fixed in the center of the lattice;
- * Auxin and Wuschel can diffuse $\rightarrow D_A, D_W$;
- * they can be degraded $\rightarrow \lambda_A, \lambda_W$;
- * they are produced through a source term with polar symmetry:

$$S(r) = \mathcal{A}_i \left(1 - \frac{r}{r_i} \right) \mathcal{H}(r_i - r),$$

$$i = A, W,$$

where:

- \mathcal{A}_i is the maximum concentration that can be produced;
- r_i is the *decay length* of the source;
- $\mathcal{H}(r_i - r)$ is the Heaviside function.



A simple diffusive model with thresholds (2)

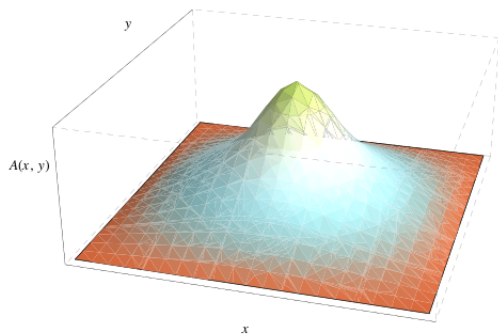
The dynamics

$$\begin{cases} \frac{\partial C_A}{\partial t} = S(r)|_{i=A} + D_A \nabla^2 C_A - \lambda_A C_A \\ \frac{\partial C_W}{\partial t} = S(r)|_{i=W} + D_W \nabla^2 C_W - \lambda_W C_W \end{cases}$$

with as BCs: $C_i(\partial \mathcal{L}) = 0, \forall i = A, W$.

How to integrate these equations?

Numerically, with a Runge–Kutta V order algorithm with adaptive step size.



A simple diffusive model with thresholds (2)

How to reproduce Floral Organ Specification?

Lattice \mathcal{L} as a set of cells: every cell can be **differentiated** OR **undifferentiated**.

differentiated \equiv gene state changes!

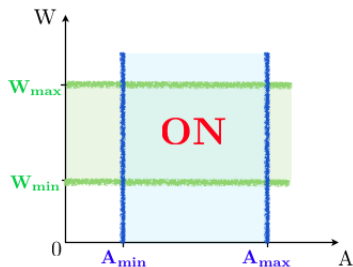
* How can we achieve this?

According to Auxin and Wuschel concentration in cells!

If this condition is satisfied, hence $A > A_{min}$ AND $W < W_{max}$, the cell becomes **differentiated** and so on...

* And the inhibitory fields?

To reproduce this, it must be remembered that Auxin's transport in cells is given by **DIFFUSION + ACTIVE TRANSPORT**.



→ the more Auxin there is, the more Auxin arrives! + Poissonian Noise on Flux

Secondments & Further Projects

About the model...

- development of sepal formation and then focus on the other organs;
- study the stochastic version of the system;
- search for adjusted parameters in the model via MCMC.



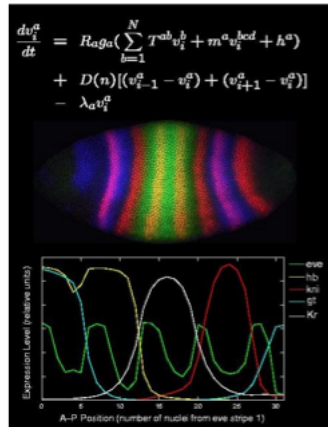
Secondments...

- *King's College of London* (UK);
- *Technische Universitaet Berlin*, Berlin, Germany.



Other Projects...

- inference on phylogenetic trees for *Influenza A* virus with Prof. Silvio Franz.



*Thank you
for your attention!*