Random Lasers and Photonic Spin-Glasses: an introduction to theoretical and experimental challenges

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in collaboration with

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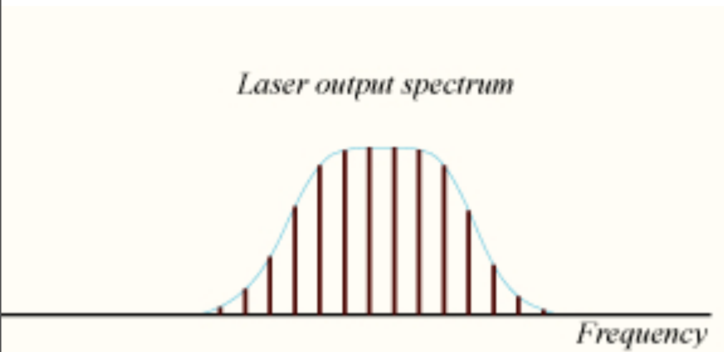
Random Laser: an old idea

Laser

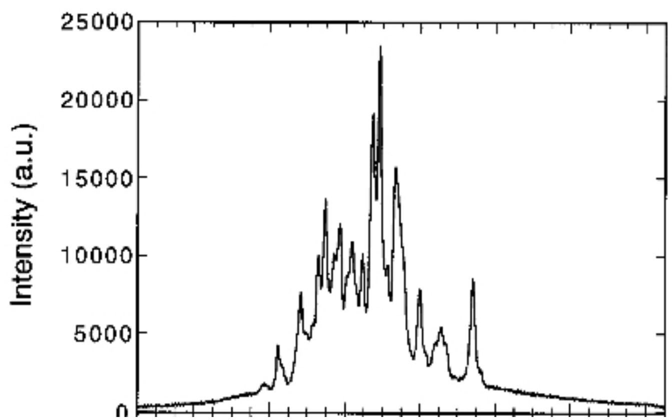
1958:
Infrared and Optical Masers, by
Arthur L. Schawlow, Charles H. Townes,
Physical Review



Random Laser

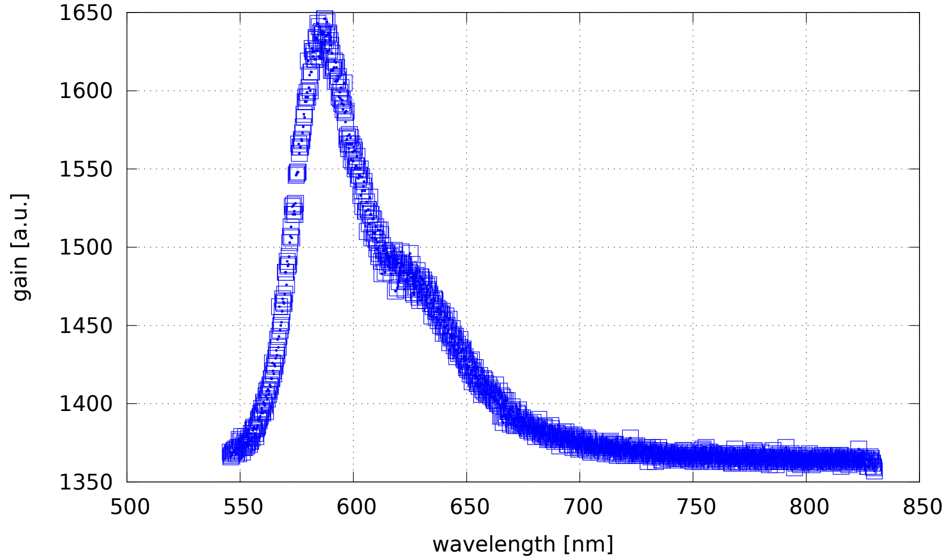
1967: VS Letokhov - JETP Lett. (theory)
Idea: Stimulated emission without a resonant cavity

1994: Lawandy
Laser action in strongly scattering media (experiments)
Realization: multiple scattering medium (i.e., set of stochastic resonators), a multimode laser with disorder.



Random Laser

Pumping mode-locked laser (pulses of high intensity)
in an optically active disordered material (powder/precipitate in light amplifying medium)



E.g., precipitate of TiO_2 particles (scatterers) in methanol solution doped by **rhodamine** (light amplifying)

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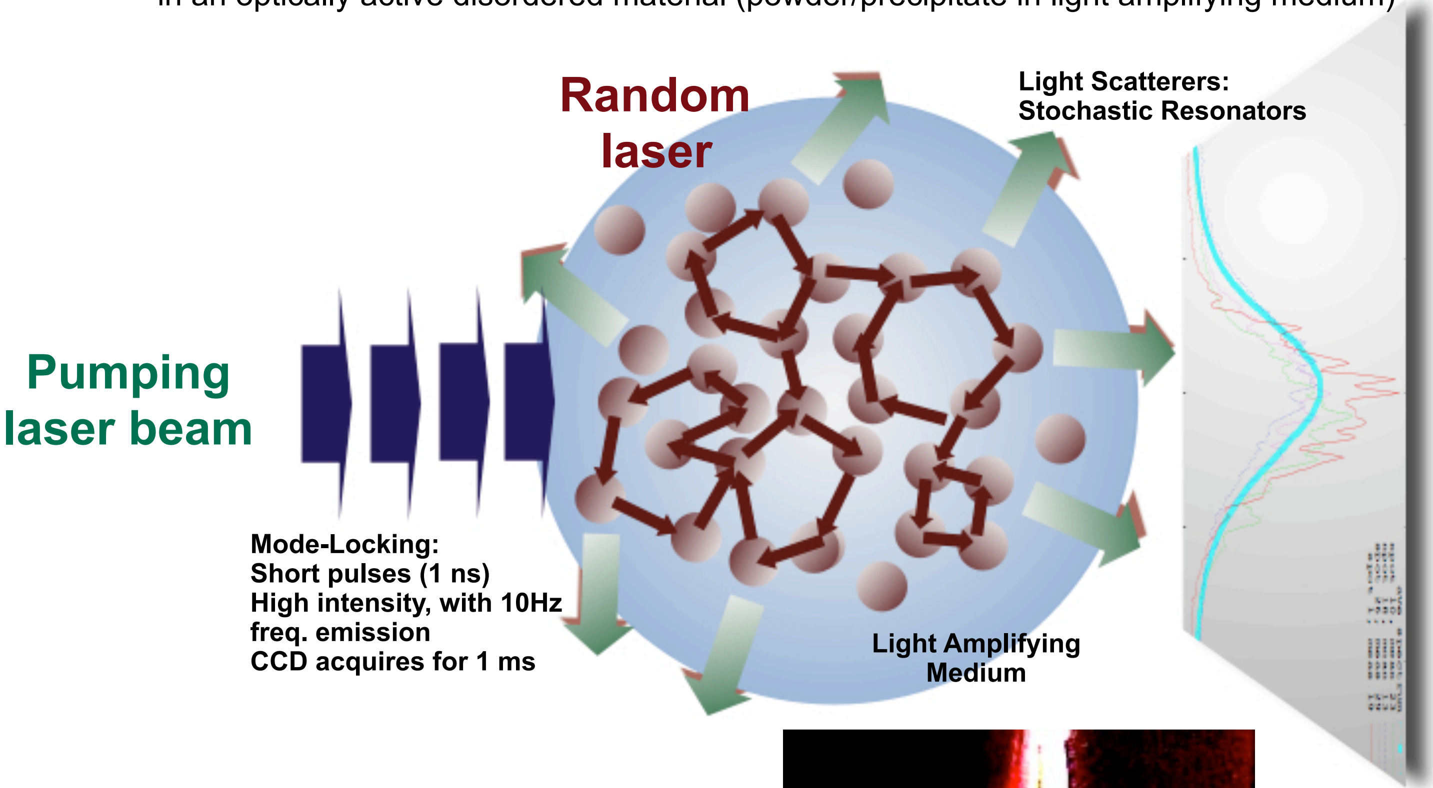

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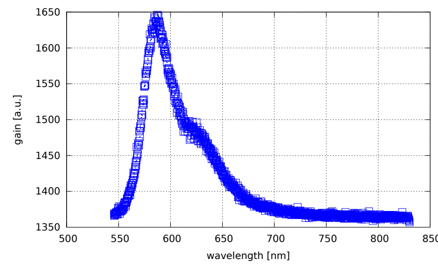
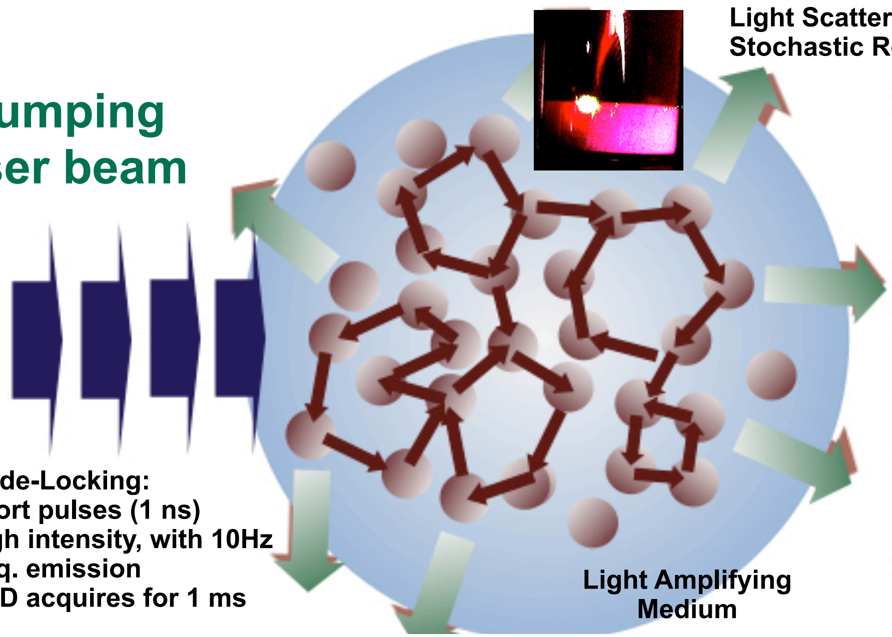
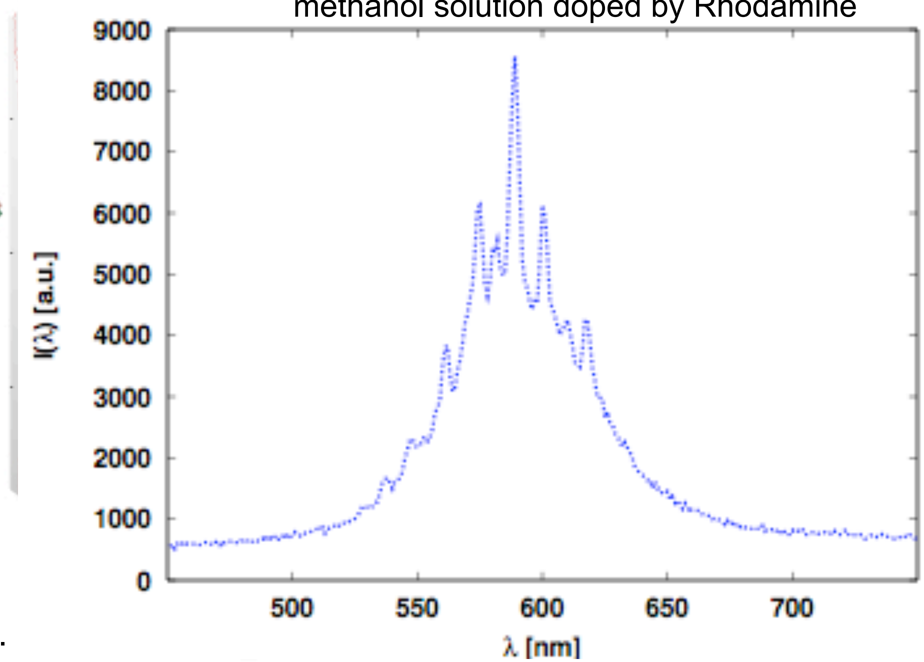


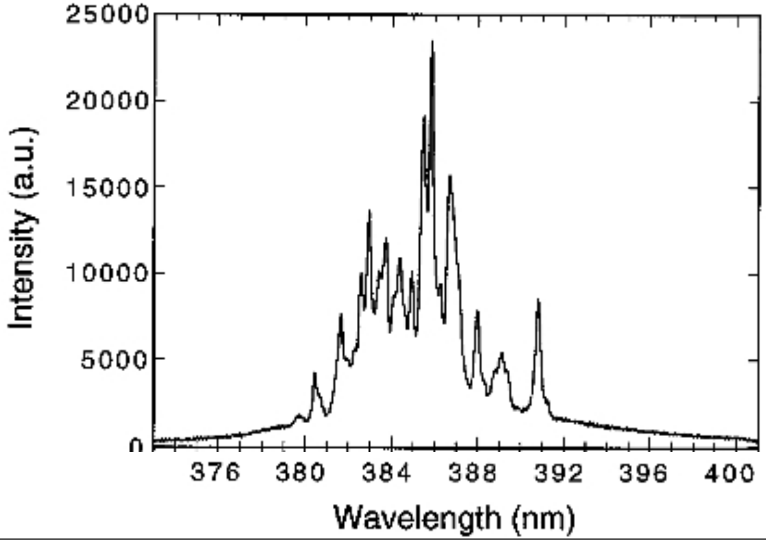
Experiments - Random Laser



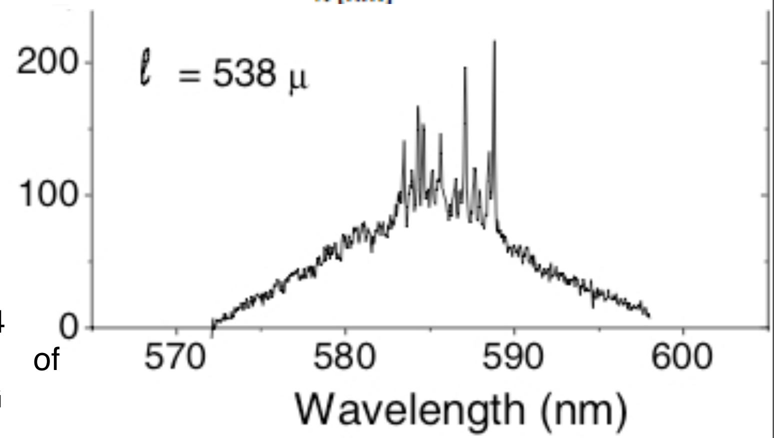
Mode-Locking:
Short pulses (1 ns)
High intensity, with 10Hz freq. emission
CCD acquires for 1 ms

M. Leonetti, LL, C. Conti, unpublished, colloidal dispersion of TiO_2 particles in methanol solution doped by Rhodamine



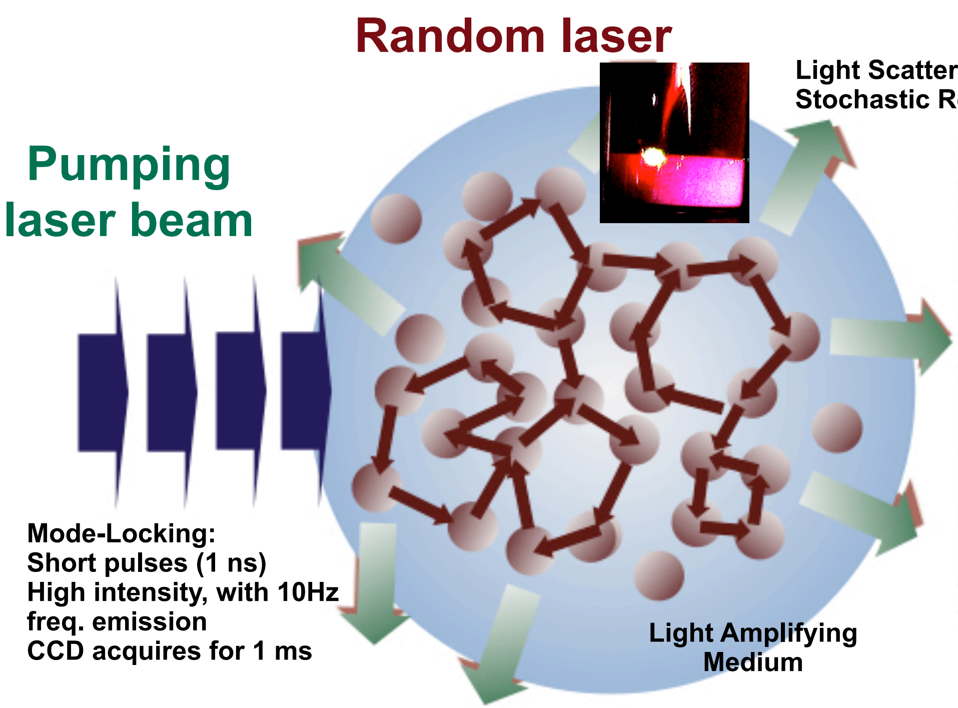


H. Cao et al.
PRL 99
ZnO powder

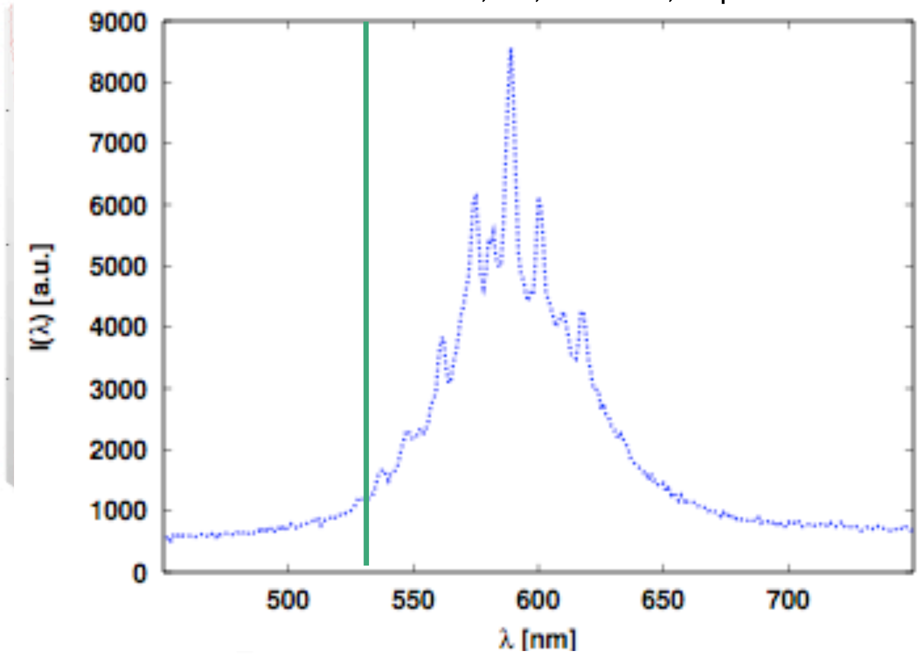


S. Mujumdar et al. PRL '04
Suspensions of ZnO in Rh6G

Experiments - Random Laser

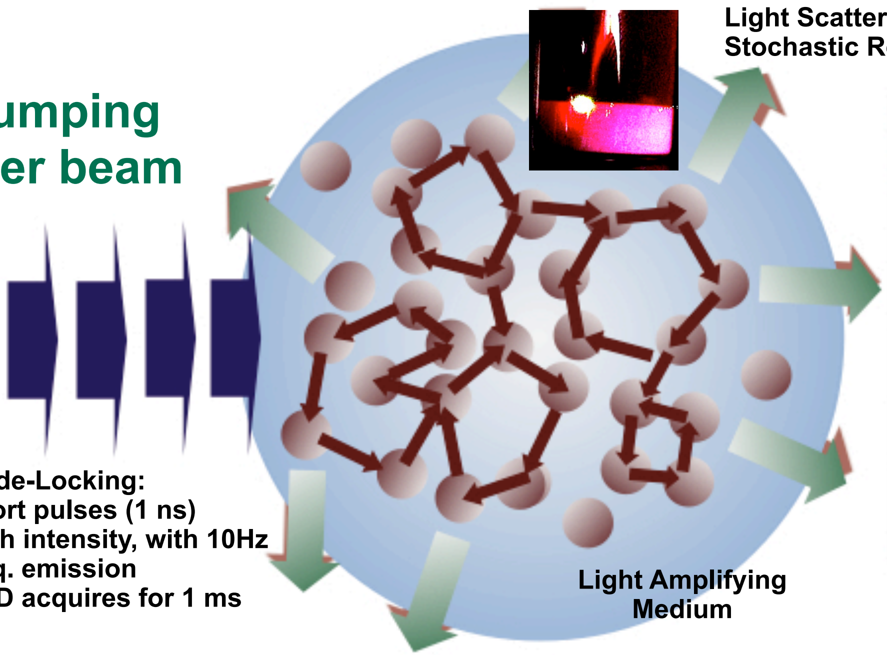


M. Leonetti, LL, C. Conti, unpublished



FAQS

Experiments - Random Laser



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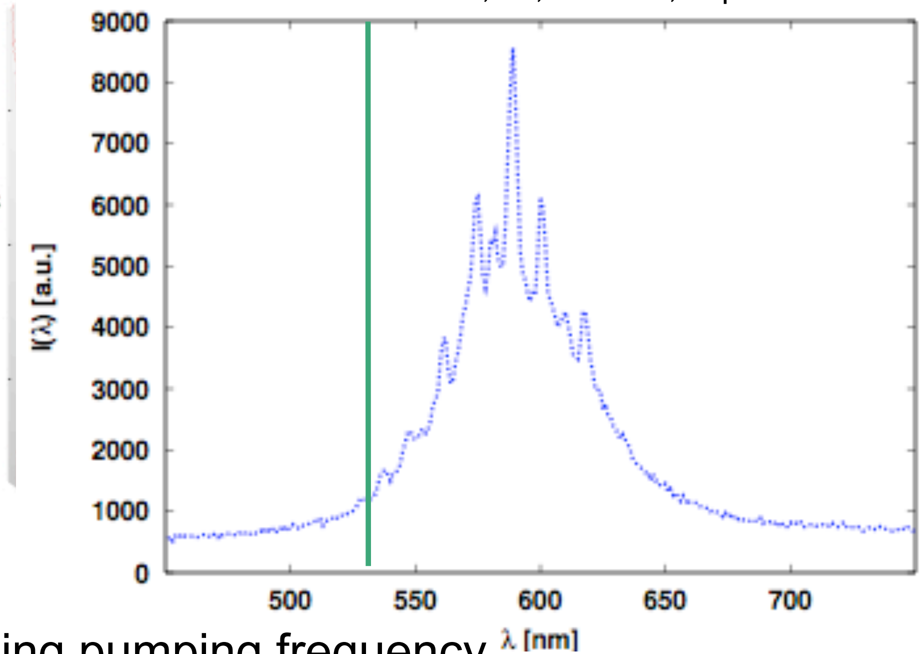
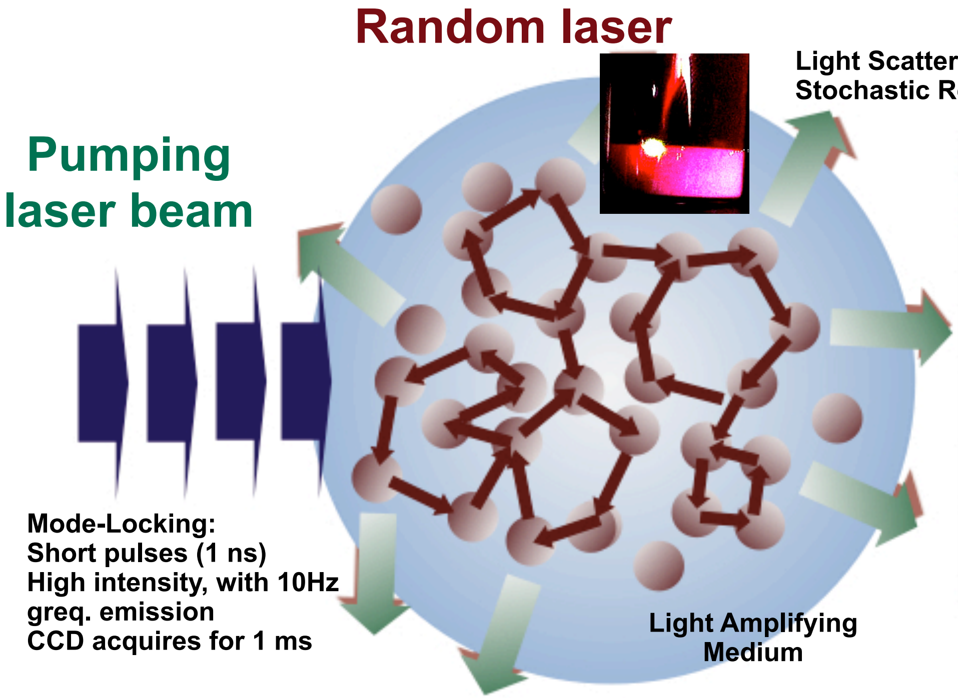
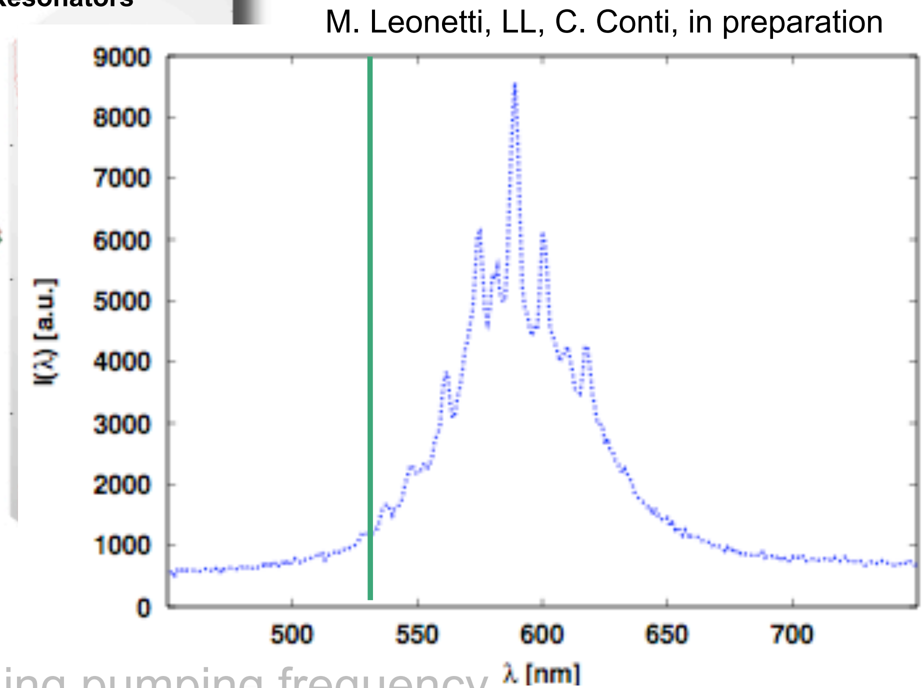


Frequency band of excited modes not including pumping frequency

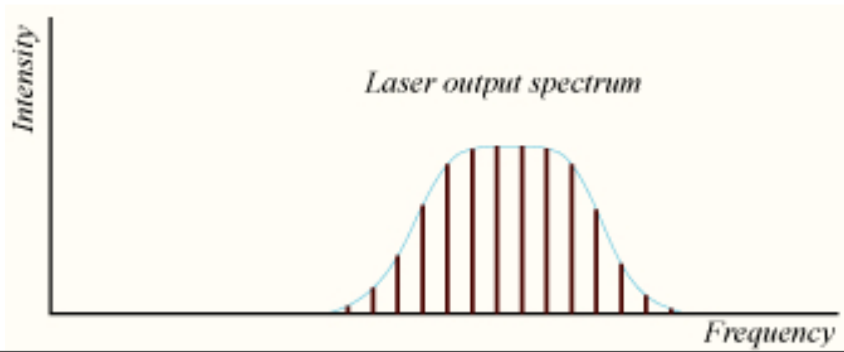
Experiments - Random Laser



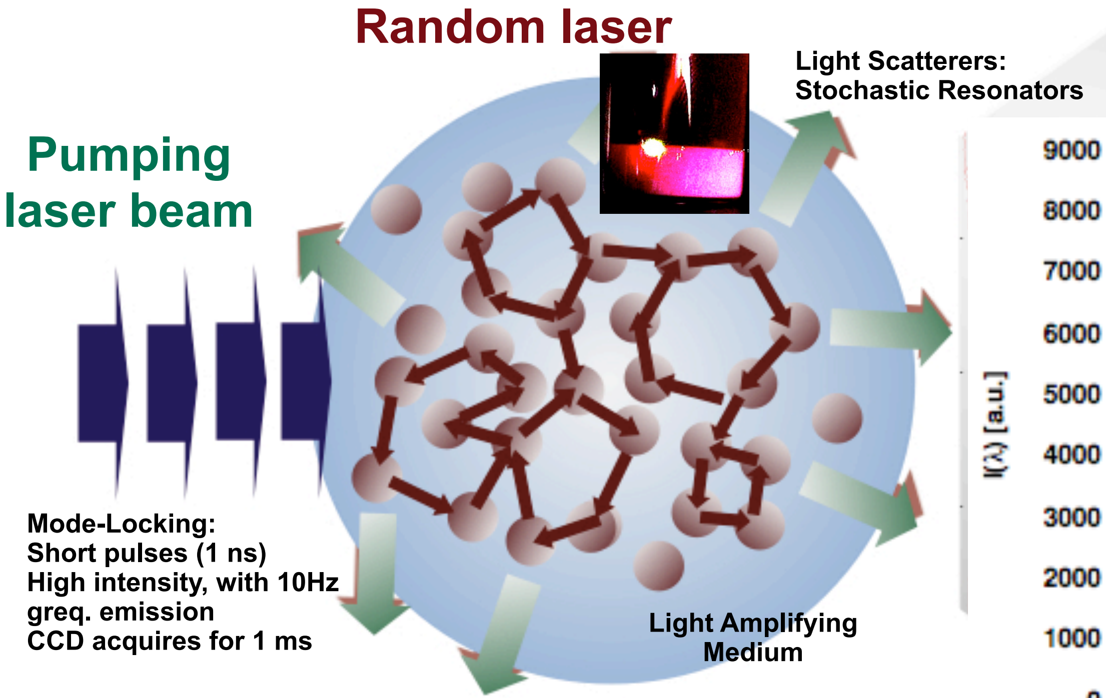


Frequency band of excited modes not including pumping frequency

Different from multimode standard laser profile



Experiments - Random Laser



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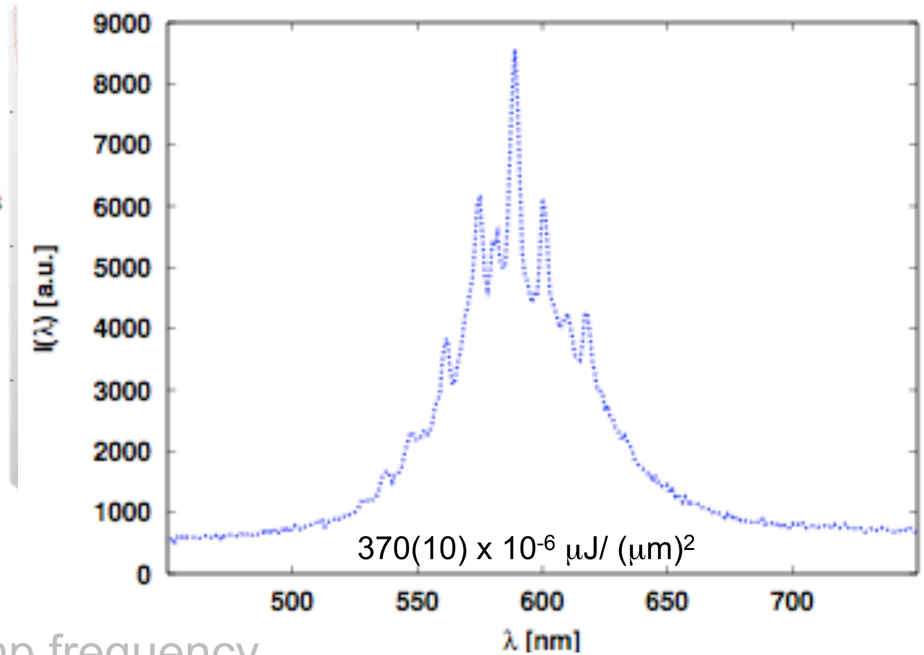
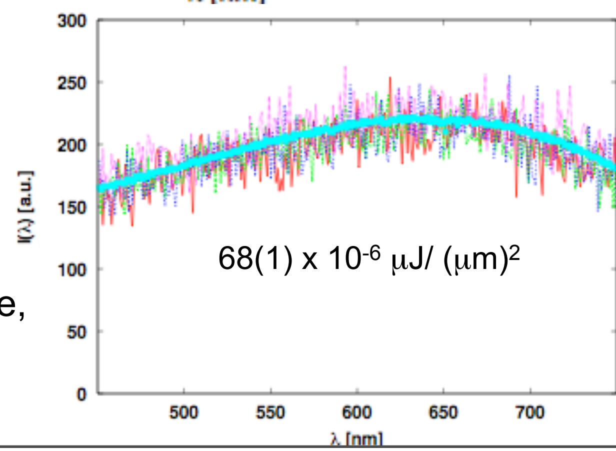


Frequency band of modes not including pump frequency

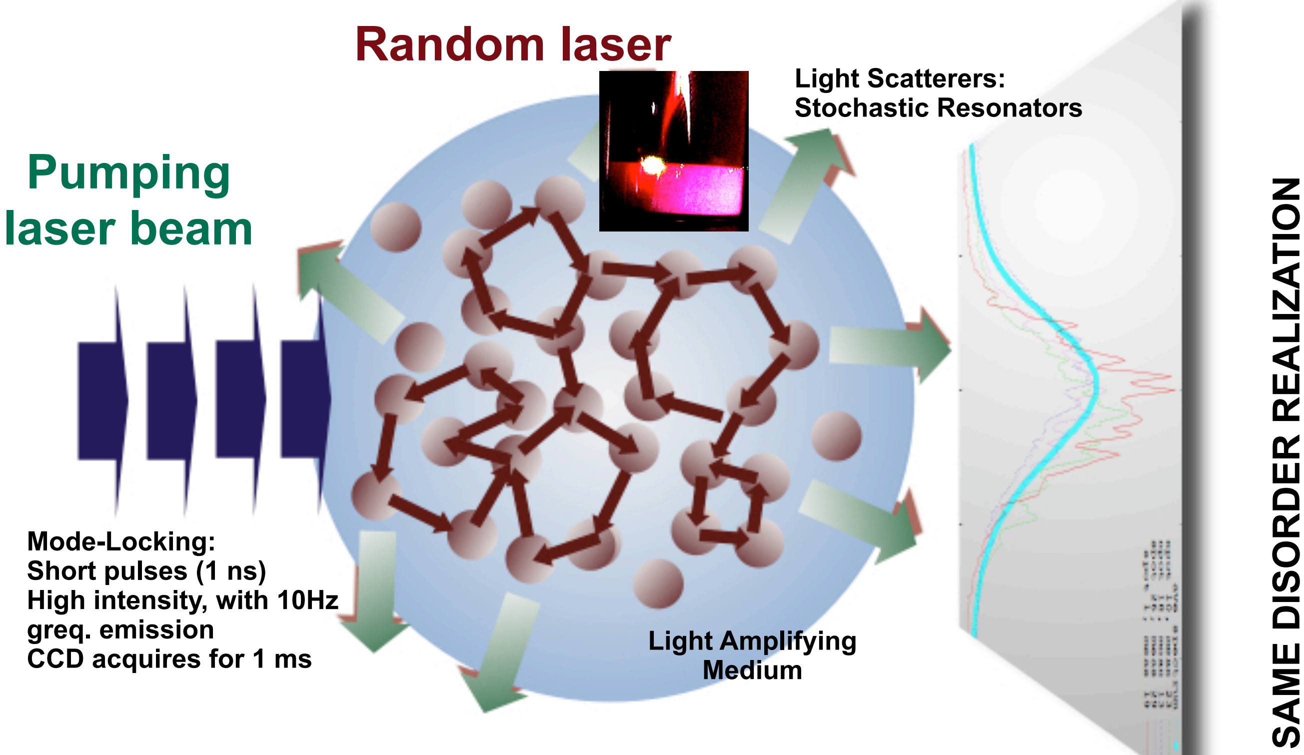
Different from multimode standard laser profile

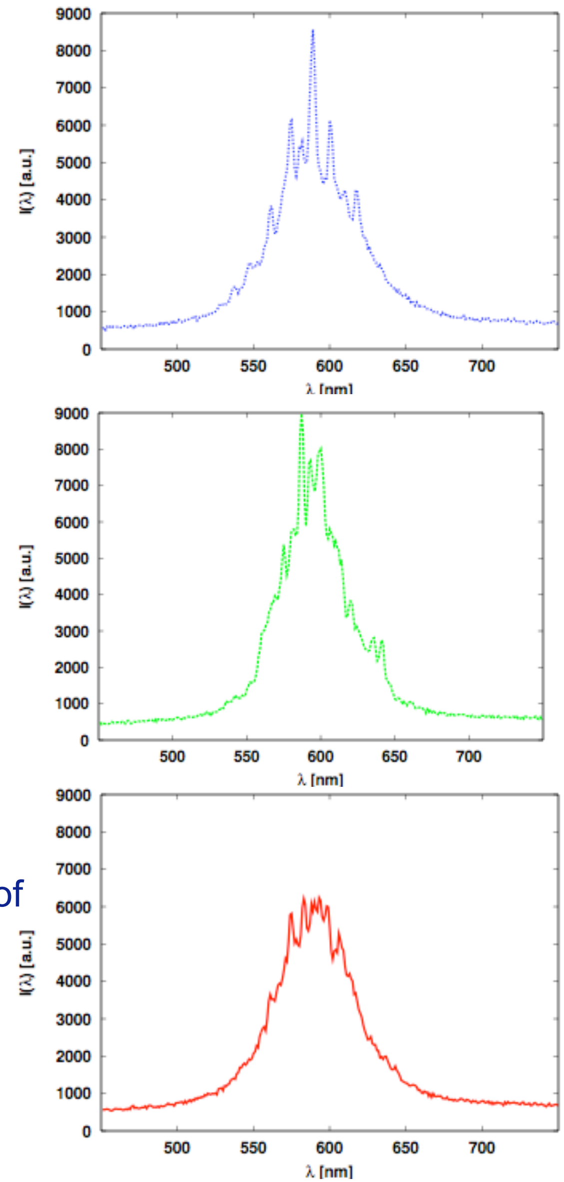
Profile typical for high pumping intensity, qualitatively different from low pumping intensity: "**transition**" from continuous wave emission to random lasing regime.

This also happens decreasing temperature, D Wiersma and S Cavalieri Nature 2001



Experiments - Random Laser



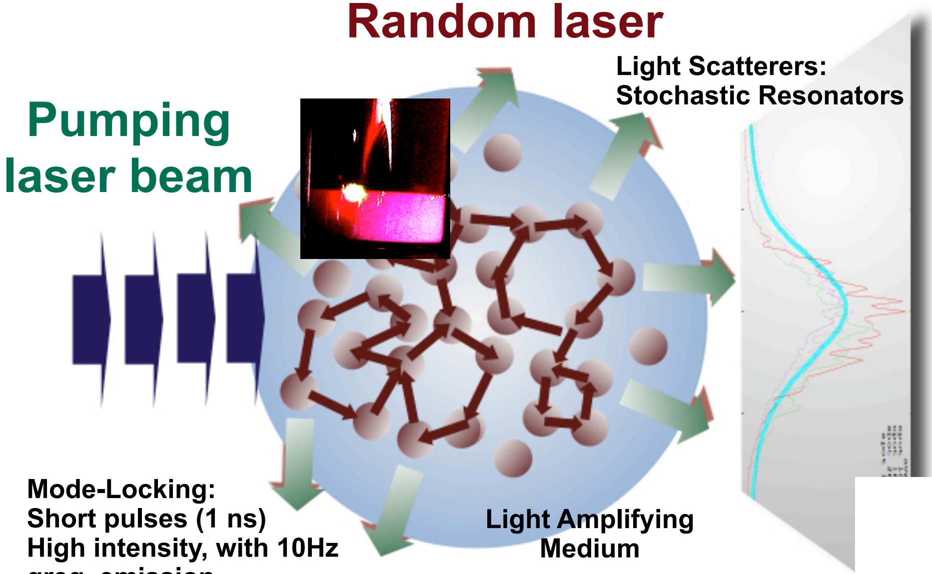


Equilibrium spectra from the **same piece of sample** (same *spot*) of random laser: different ensembles of modes activated on each experimental measurement.

M. Leonetti, LL, C. Conti, unpublished

Experiments - Random Laser

Random laser



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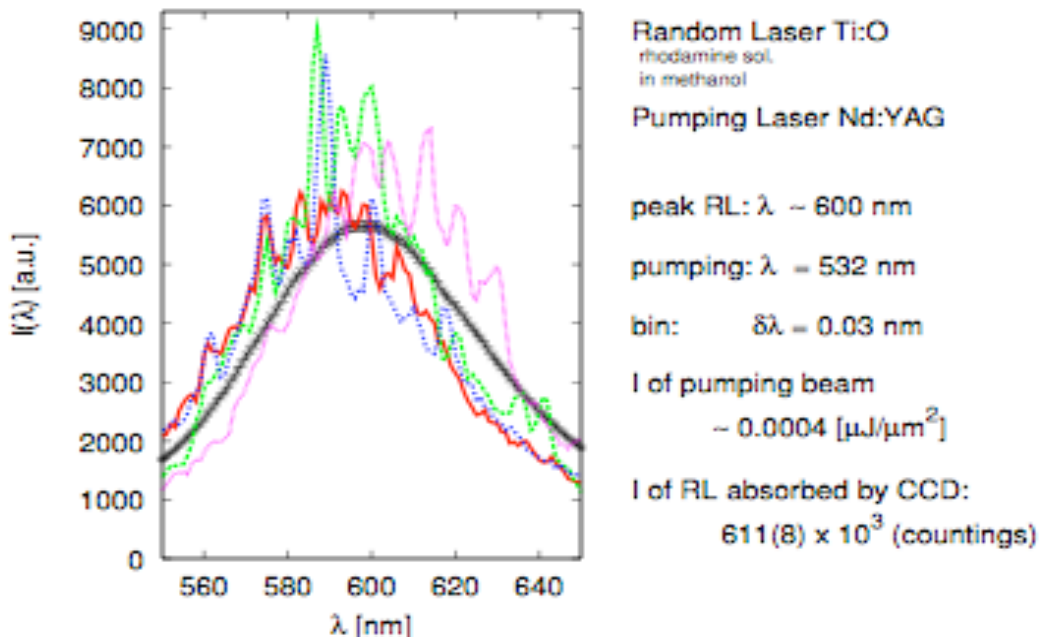
Equilibrium spectra from the **same piece of sample** (same *spot*) of random laser: different ensembles of modes activated on each experimental measurement.

Explanation:
Chaos?
Experimental uncertainty?



M. Leonetti, LL, C. Conti, unpublished

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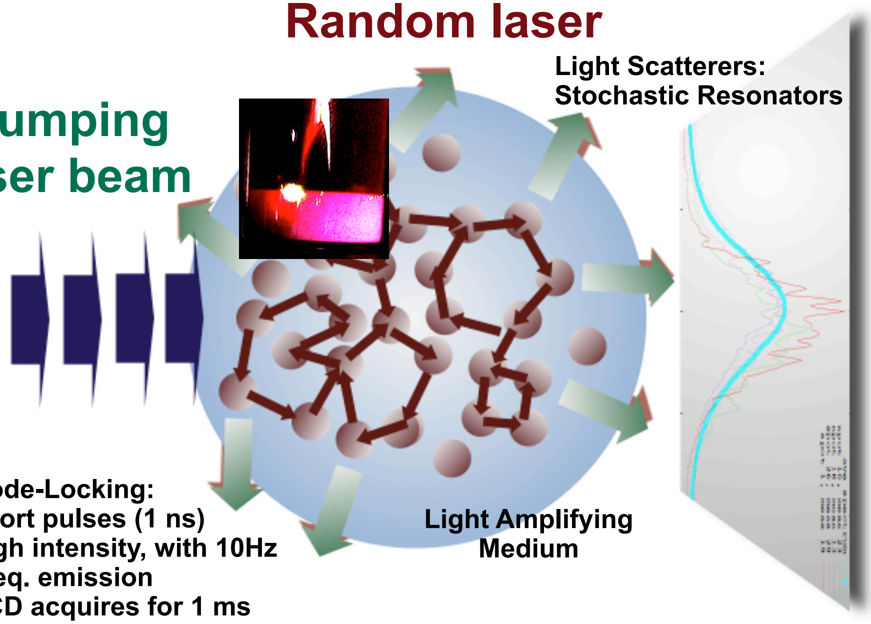
Experiments - Random Laser

Random laser

Pumping laser beam



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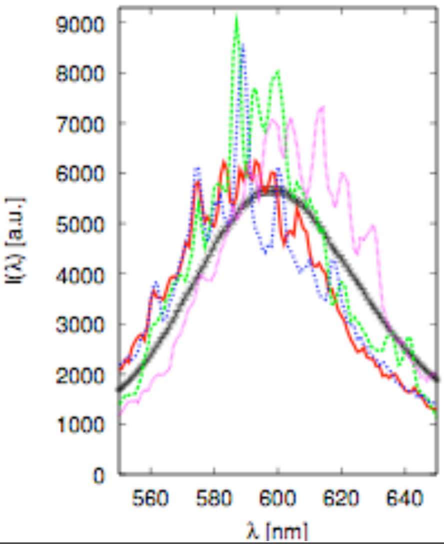


Random Laser Ti:O
rhodamine sol.
in methanol

Pumping Laser Nd:YAG

peak RL: λ ~ 600 nm

pumping: λ = 532 nm

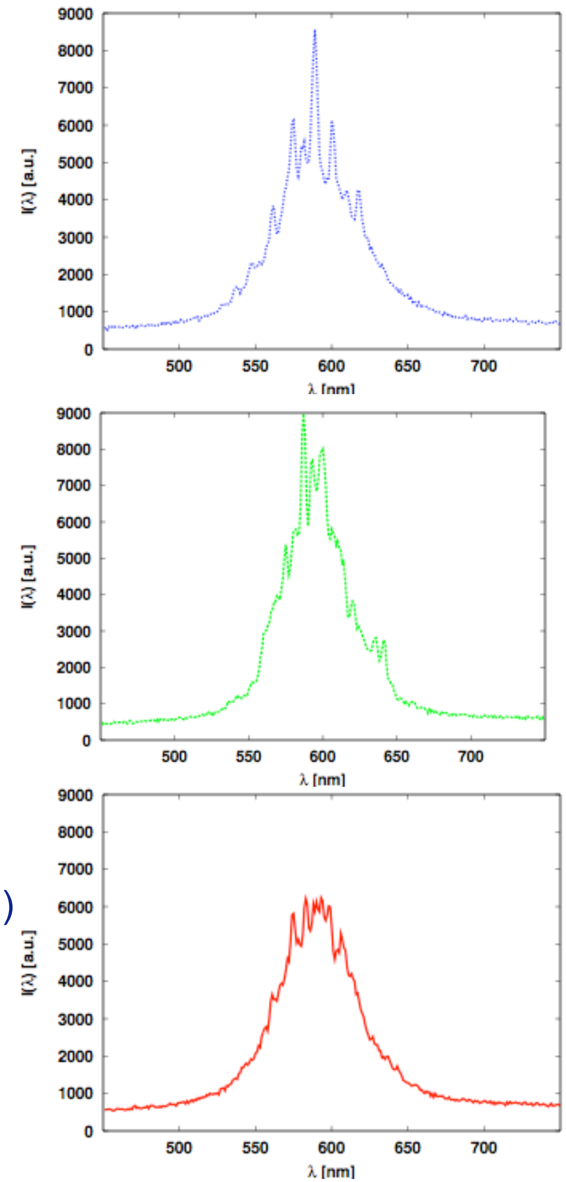
bin: δλ = 0.03 nm

I of pumping beam
~ 0.0004 [μJ/μm²]

I of RL absorbed by CCD:
611(8) x 10³ (countings)

Explanation (proposal):
Mode competition phenomena
(frequency locking at each measure)
Conjecture:
Complex free energy landscape

SAME DISORDER REALIZATION



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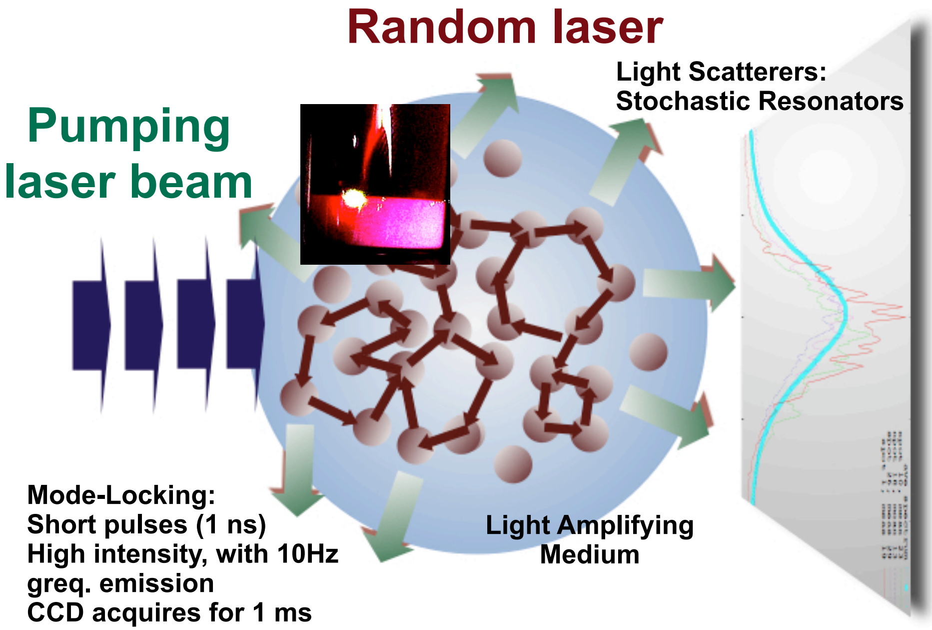
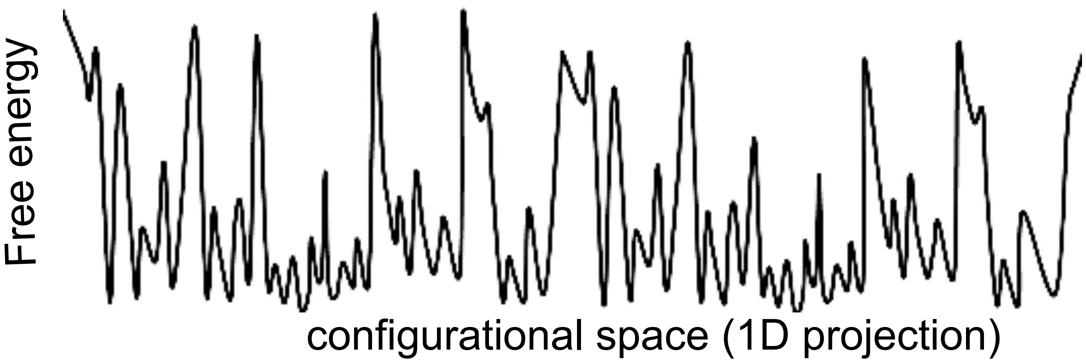
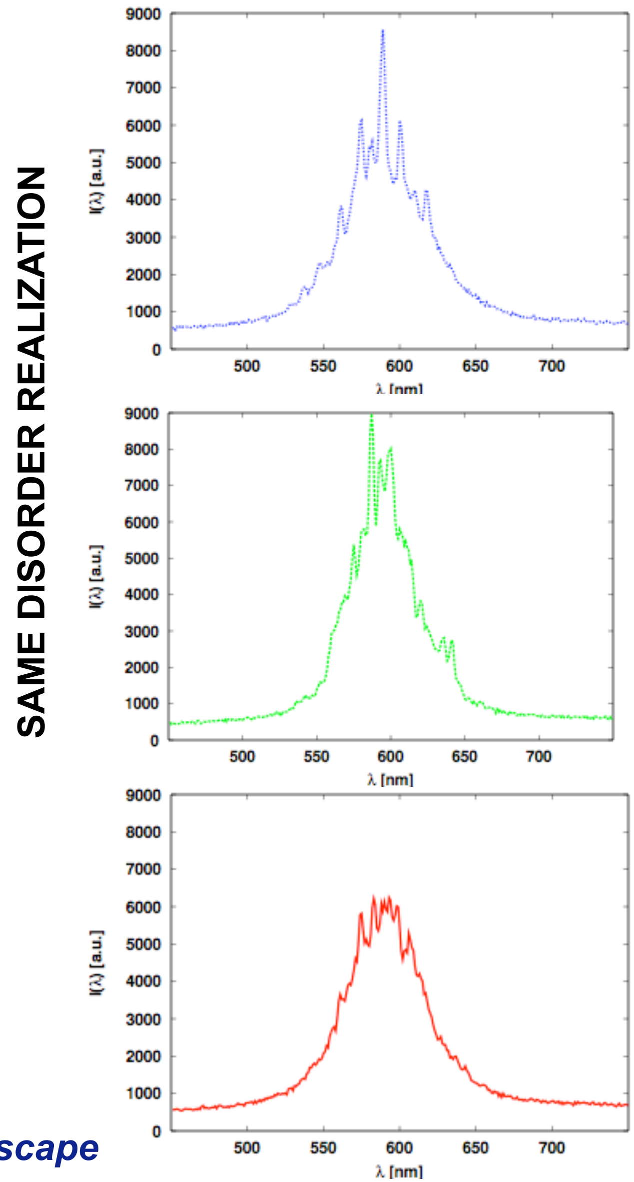
Tuesday, February 12, 2013

Experiments - Random Laser







Complex free energy landscape

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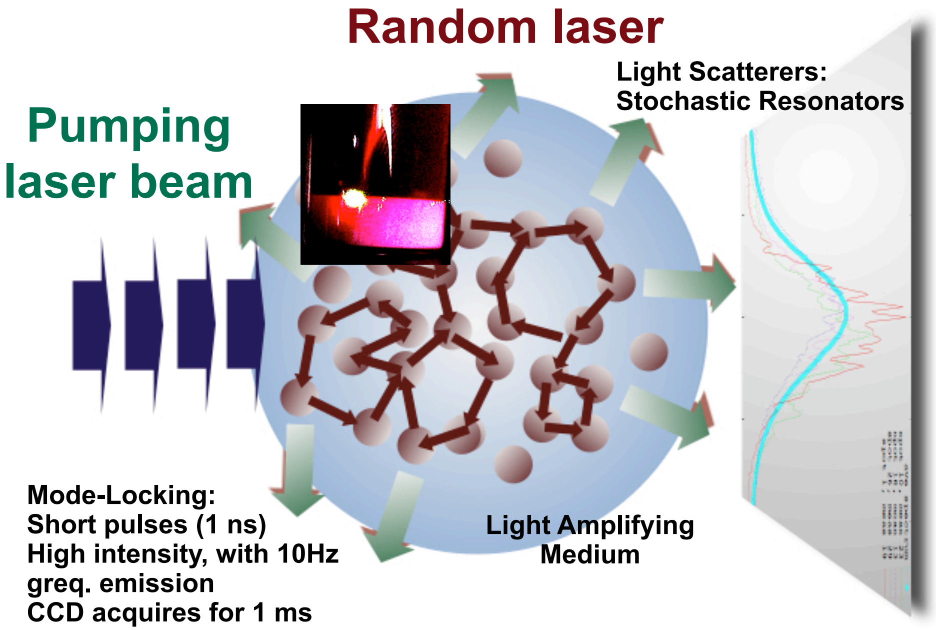
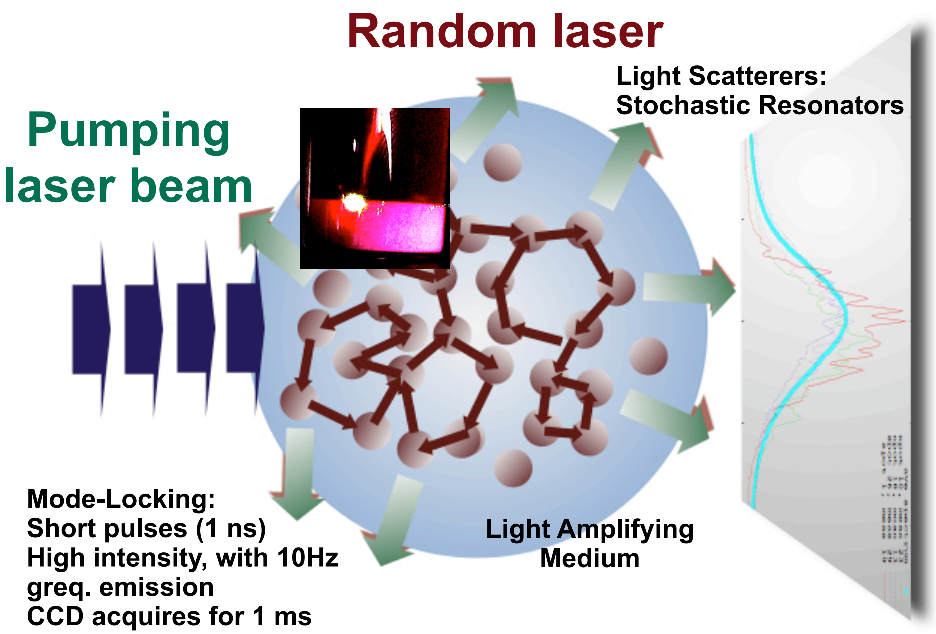
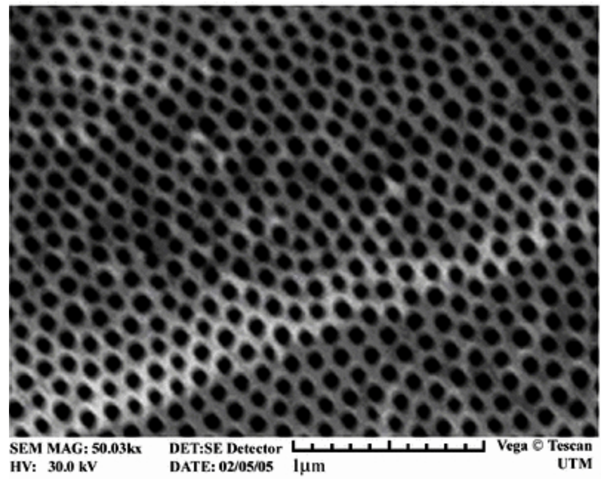


This is not general for all random lasers

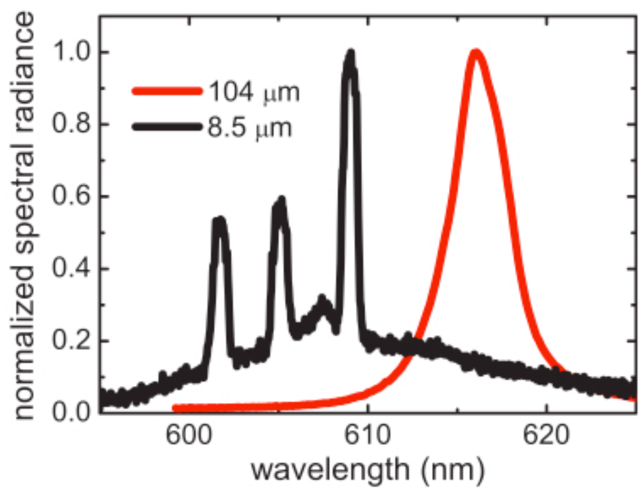
Experiments - Random Laser





Porous GaP

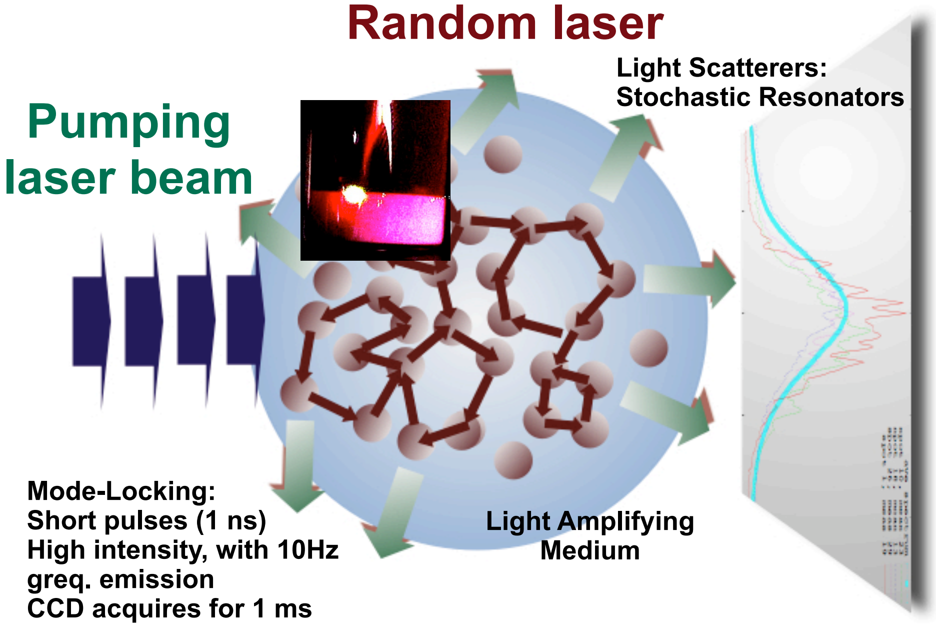
This is not general for all random lasers

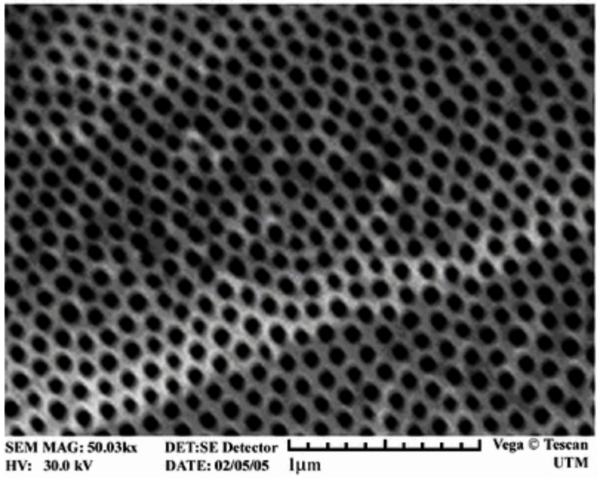


There are also random lasing materials [porous semiconducting matrix infiltrated with and embedded in laser dye] where reproducibility of spikes is claimed.

El-Dardiry *et al.* Phys. Rev. A 2010

Experiments - Random Laser





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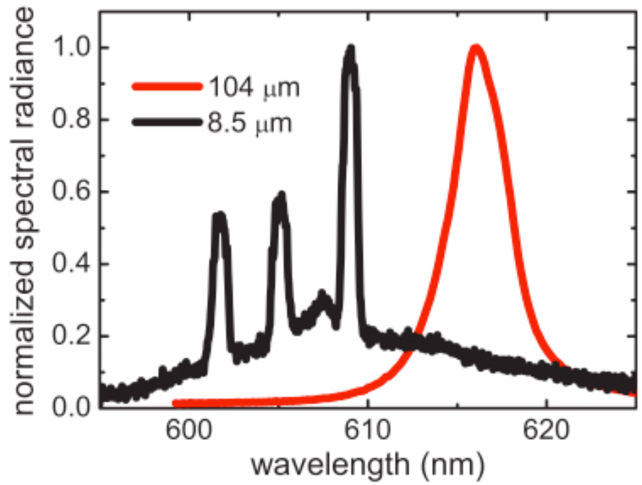

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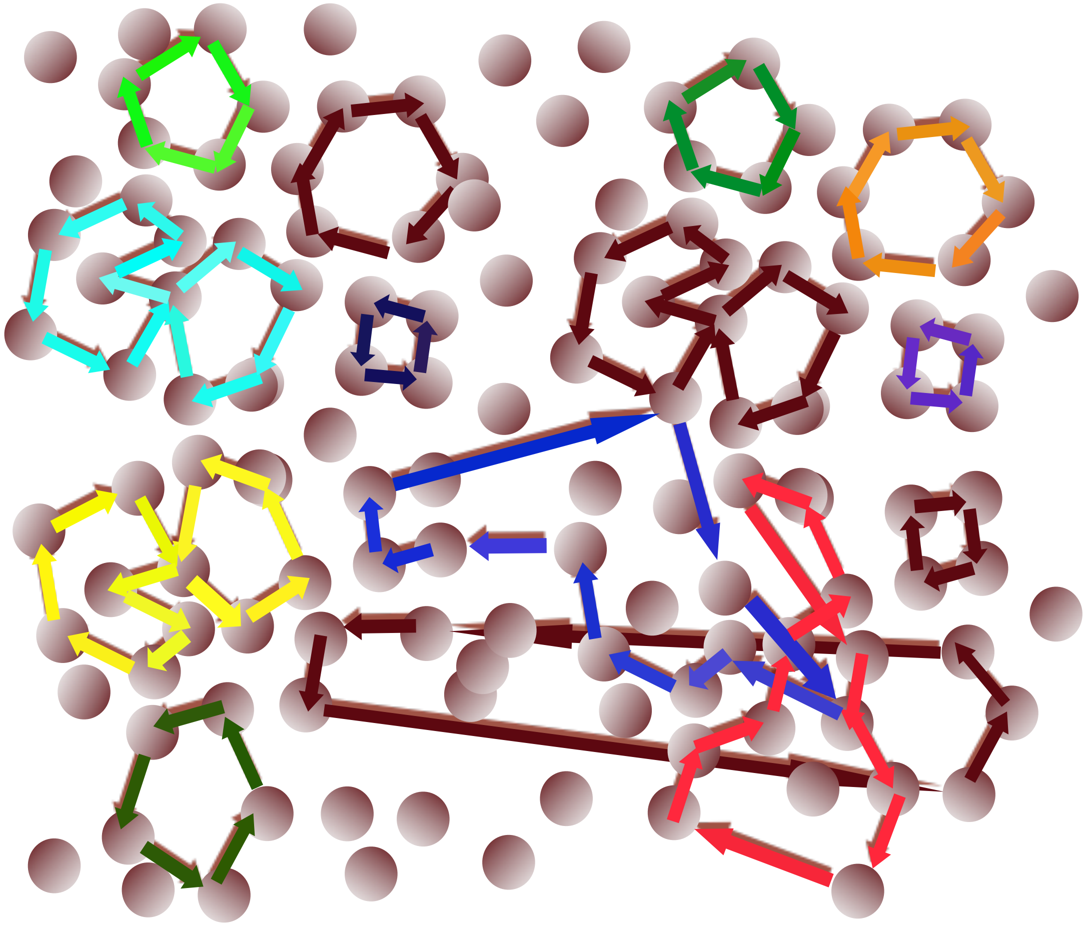
cavity-less laser?

Modeling random laser modes in space:
the electromagnetic field of each localized light mode of frequency ω is non-zero in a given region of space **r.**



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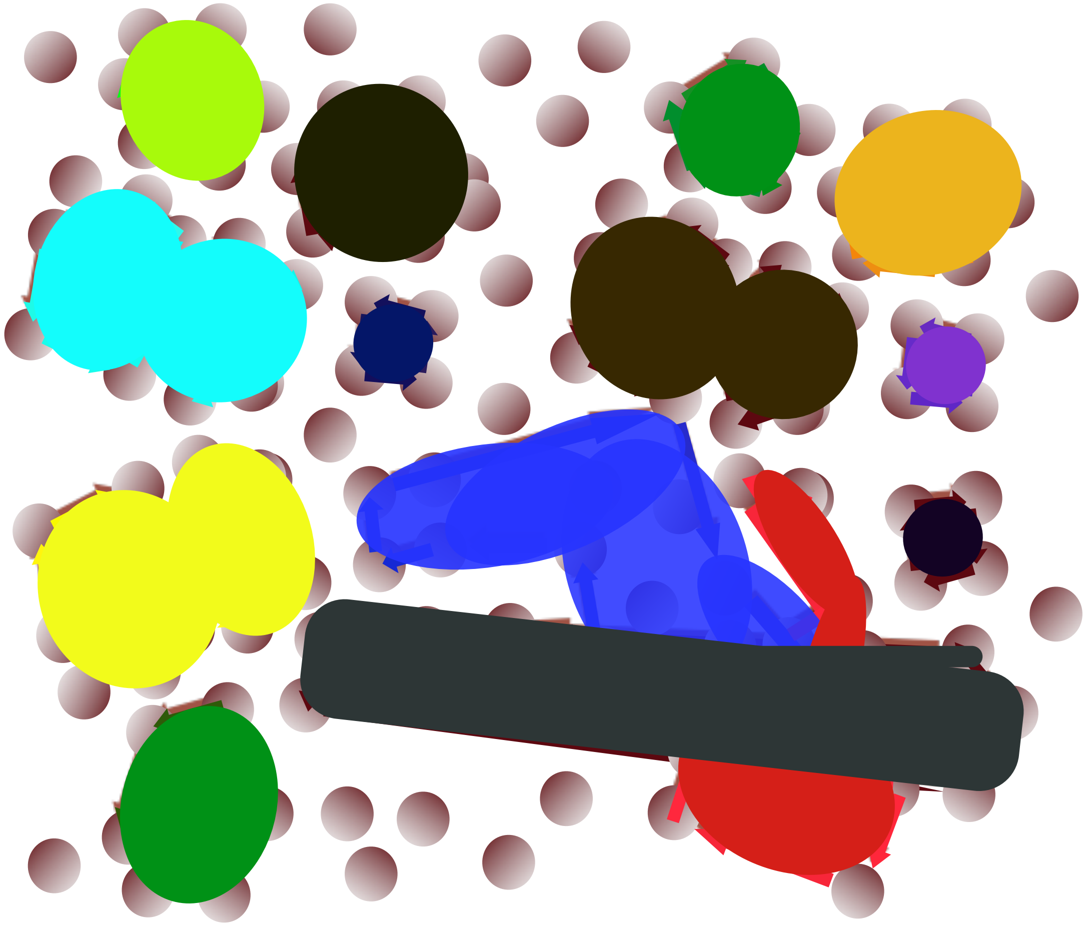

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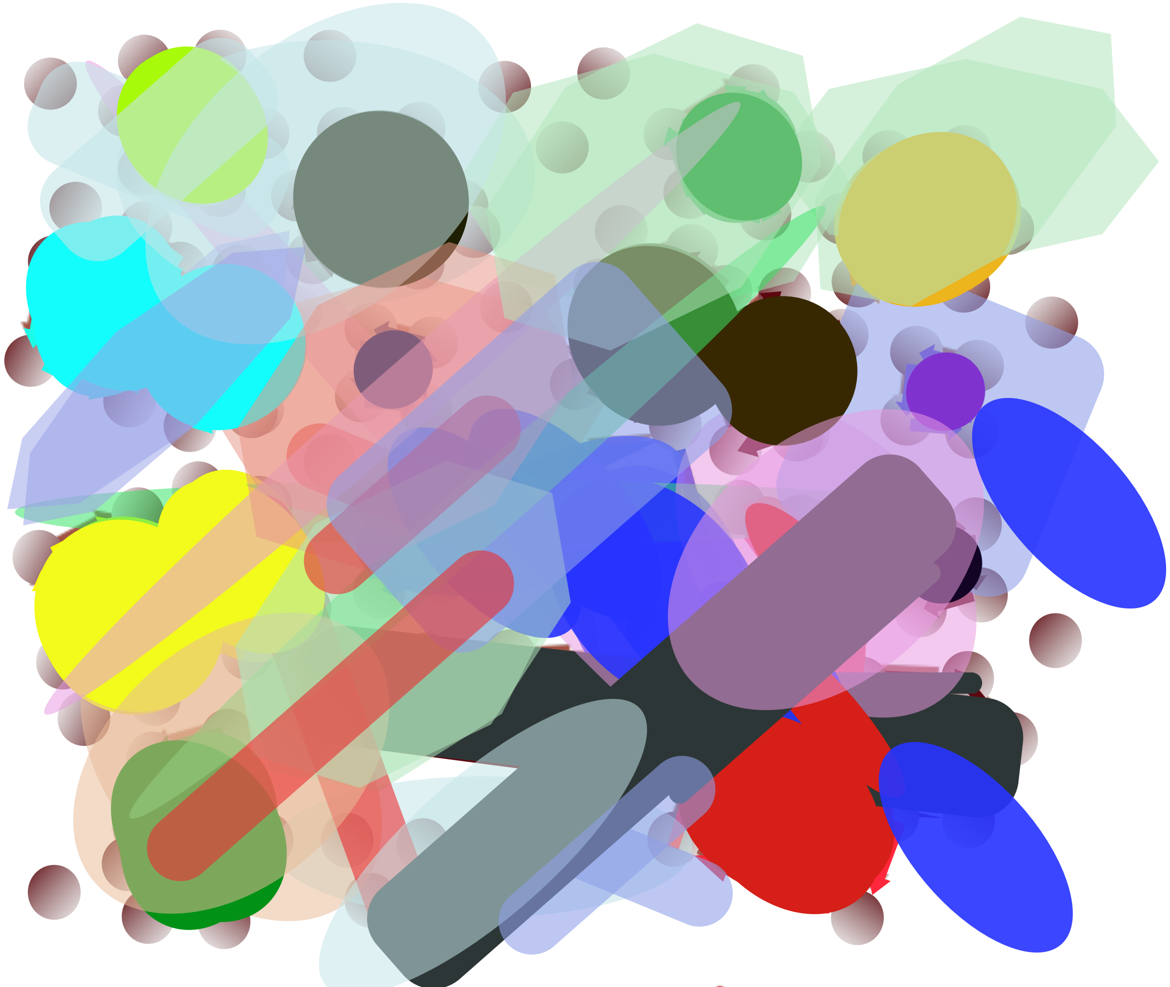

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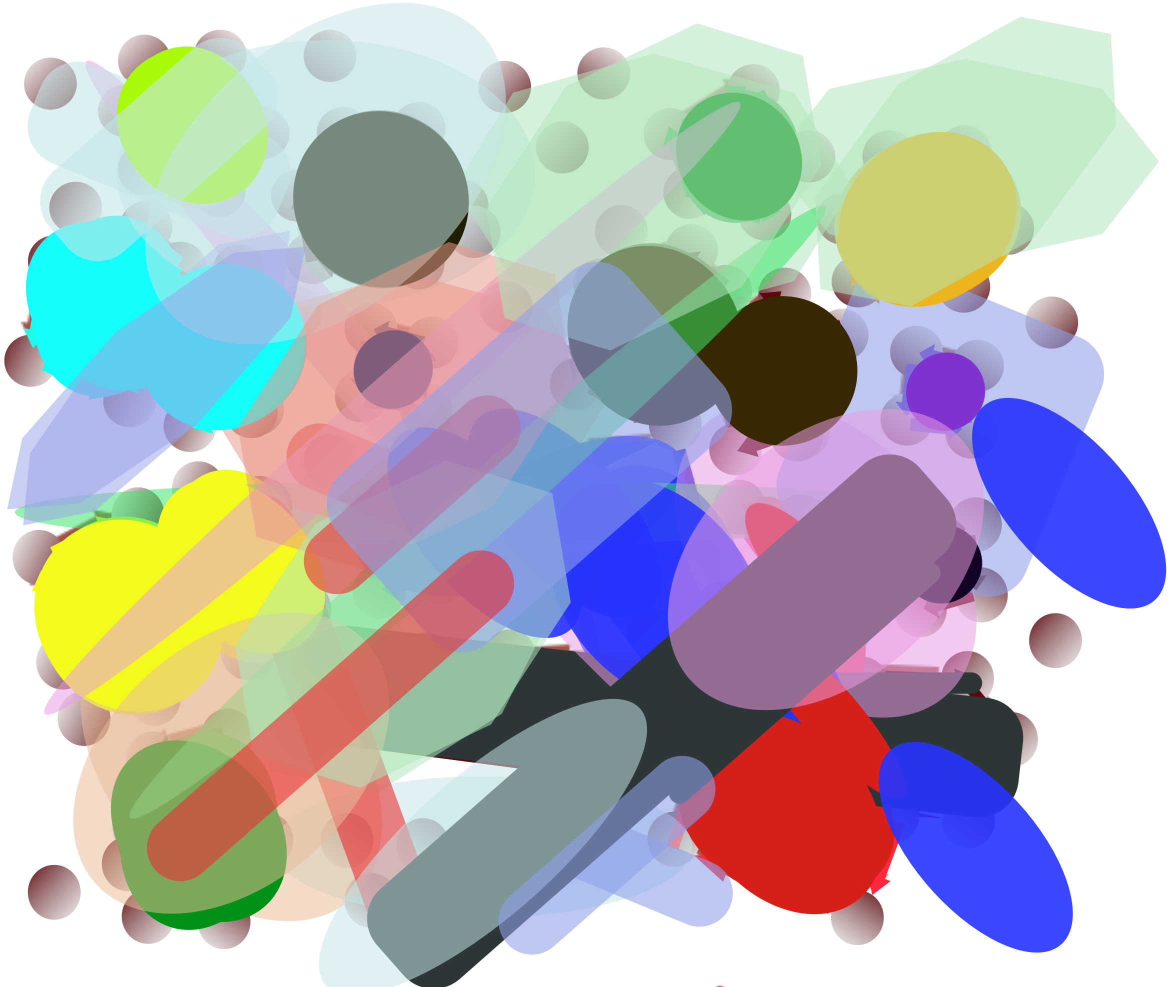

A bit less pictorial...?

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Theory - Coupled Light Modes

Electromagnetic Cavity of refractive index profile $n(\mathbf{r})$

non linear polarization $\mathbf{P}_{NL}(\mathbf{r})$

displacement vector $\mathbf{D}(\mathbf{r}) = \epsilon_0 n^2(\mathbf{r})\mathbf{E}(\mathbf{r}) + \mathbf{P}_{NL}(\mathbf{r})$

Maxwell equations in presence of nonlinear polarization in an electromagnetic cavity

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \epsilon_0 n^2(\mathbf{r})\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}_{NL}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\mu_0 \partial_t \mathbf{H}$$

K Sakoda, *Optical Properties of Photonic Crystals, 2001*
HA Haus, *Waves and Fields in Optoelectonics, 1984*
L Angelani et al. PRB 06

Solution to the equations is a superposition of modes:

$$\mathbf{E} = \mathrm{Re}\ \left[\sum_n a_n(t)\mathbf{E}_n(\mathbf{r}) \exp(-i\omega_n t)\right]$$

$$\mathbf{H} = \mathrm{Re}\ \left[\sum_n a_n(t)\mathbf{H}_n(\mathbf{r}) \exp(-i\omega_n t)\right]$$

Complex amplitudes such that total energy stored in the EM cavity (closed):

$$\mathcal{E} = \sum_k \mathcal{E}_k = \sum_k |a_k|^2$$

Theory - Coupled Light Modes

Electromagnetic Cavity of refractive index profile $n(\mathbf{r})$
non linear polarization $\mathbf{P}_{NL}$ and displacement vector $\mathbf{D}(\mathbf{r}) = \epsilon_0 n^2(\mathbf{r})\mathbf{E}(\mathbf{r}) + \mathbf{P}_{NL}(\mathbf{r})$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \epsilon_0 n^2(\mathbf{r})\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}_{NL}}{\partial t} \qquad \nabla \times \mathbf{E} = -\mu_0 \partial_t \mathbf{H}$$

$$\mathbf{E} = \mathrm{Re}\ \left[\sum_n a_n(t)\mathbf{E}_n(\mathbf{r}) \exp(-i\omega_n t)\right]$$

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$$\mathcal{E} = \sum_k \mathcal{E}_k = \sum_k |a_k|^2$$

If $\mathbf{P}_{NL}$=0, amplitudes are constant the whole time dependence is in the oscillation and $\mathbf{E}_n$, $\mathbf{H}_n$ are the eigenvalues of the system

$$\mathcal{L}\mathcal{F}_n^{(0)} = \omega_n \mathcal{M}\mathcal{F}_n^{(0)} \qquad \mathcal{F}_n^{(k)} = \begin{pmatrix} \mathbf{E}_n^{(k)} \\ \mathbf{H}_n^{(k)} \end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix} 0 & i\nabla\times \\ -i\nabla\times & 0 \end{pmatrix} \qquad \mathcal{M} = \begin{pmatrix} \epsilon_0 n^2(\mathbf{r}) & 0 \\ 0 & \mu_0 \end{pmatrix}$$

Theory - Coupled Light Modes

Electromagnetic Cavity of refractive index profile $n(\mathbf{r})$
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$$\mathbf{E} = \mathrm{Re}\ \left[\sum_n a_n(t)\mathbf{E}_n(\mathbf{r})\exp(-i\omega_n t)\right]$$

$$\mathcal{E} = \sum_k \mathcal{E}_k = \sum_k |a_k|^2$$

$$\mathbf{H} = \mathrm{Re}\ \left[\sum_n a_n(t)\mathbf{H}_n(\mathbf{r})\exp(-i\omega_n t)\right]$$

If **P**$_{NL}$ is not zero, amplitudes depend on time and solution has the general form above
To obtain the time evolution of the amplitudes we expand around weak polarization, i.e., slowly varying amplitudes: multiscale approach

$$a(t) = a(t_1, t_2, \ldots, t_n)$$

$$t_n = \eta^n t \qquad t_0 = t$$

$$\partial_t = \sum_k \frac{\partial t_k}{\partial t}\partial_{t_k} = \sum_k \eta^k \partial_{t_k} \simeq \partial_{t_0} + \eta\partial_{t_1} + \ldots$$

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Theory - Coupled Light Modes

If $\mathbf{P}_{NL}$ is not zero, amplitudes depend on time and solution has the general form above
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$$\mathbf{E} \simeq \mathrm{Re}\left\{\sum_n \left[\mathbf{E}_n^{(0)} + \eta\mathbf{E}_n^{(1)} + \ldots\right] \exp(-\imath\omega_n t)\right\}$$

$$\mathbf{H} \simeq \mathrm{Re}\left\{\sum_n \left[\mathbf{H}_n^{(0)} + \eta\mathbf{H}_n^{(1)} + \ldots\right] \exp(-\imath\omega_n t)\right\}$$

$$t_n = \eta^n t$$

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$$t_n = \eta^n t$$

$$\mathbf{P}_{NL} = \mathrm{Re}\left[\sum_n \mathbf{P}_n(t_1, t_2, \ldots) \exp(-\imath\omega_n t)\right]$$

$$\simeq \mathrm{Re}\left[\sum_n \left(\mathbf{P}_n^{(0)} + \eta \mathbf{P}_n^{(1)} + \ldots\right) \exp(-\imath\omega_n t)\right]$$

$$\mathbf{J} = \partial_t \mathbf{P}_{NL} = \mathrm{Re}\left[\sum_n \mathbf{J}_n(t_1, t_2, \ldots) \exp(-\imath\omega_n t)\right]$$

$$\simeq \mathrm{Re}\left[\sum_n \left(\mathbf{J}_n^{(0)} + \eta \mathbf{J}_n^{(1)} + \ldots\right) \exp(-\imath\omega_n t)\right]$$

$$\mathbf{J}_n^{(0)} = -i\omega_n \mathbf{P}_n^{(0)}$$

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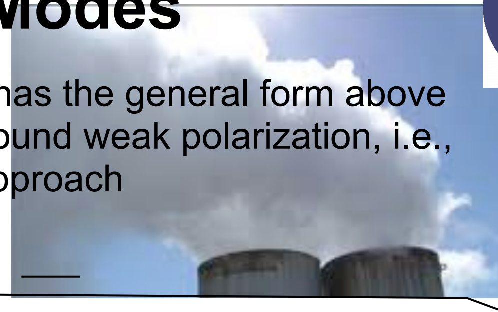
Theory - Coupled Light Modes



If **PNL** is not zero, amplitudes depend on time and solution has the general form above
To obtain the time evolution of the amplitudes we expand around weak polarization, i.e., slowly varying amplitudes: multi-scale approach

$$\mathbf{J} = \partial_t \mathbf{P}_{NL} = \mathrm{Re}\left[\sum_n \mathbf{J}_n(t_1, t_2, \ldots) \exp(-\imath\omega_n t)\right]$$

$$\simeq \mathrm{Re}\left[\sum_n \left(\mathbf{J}_n^{(0)} + \eta \mathbf{J}_n^{(1)} + \ldots\right) \exp(-\imath\omega_n t)\right]$$

$$\mathbf{E} \simeq \mathrm{Re}\left\{\sum_n \left[\mathbf{E}_n^{(0)} + \eta \mathbf{E}_n^{(1)} + \ldots\right] \exp(-\imath\omega_n t)\right\}$$

$$\mathbf{H} \simeq \mathrm{Re}\left\{\sum_n \left[\mathbf{H}_n^{(0)} + \eta \mathbf{H}_n^{(1)} + \ldots\right] \exp(-\imath\omega_n t)\right\}$$

$$\mathbf{P}_{NL} = \mathrm{Re}\left[\sum_n \mathbf{P}_n(t_1, t_2, \ldots) \exp(-\imath\omega_n t)\right]$$

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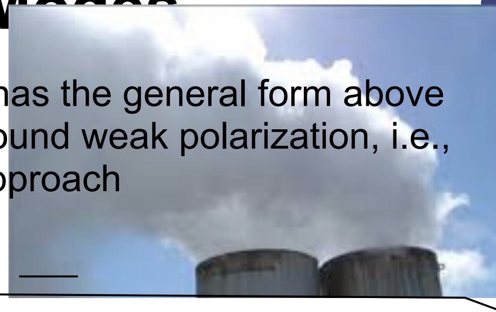
$$\mathbf{J}_n^{(0)} = -i\omega_n \mathbf{P}_n^{(0)}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \epsilon_0 n^2(\mathbf{r}) \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}_{NL}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\mu_0 \partial_t \mathbf{H}$$

$$t_n = \eta^n t$$

Theory - Coupled Light Modes



If **PNL** is not zero, amplitudes depend on time and solution has the general form above
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$$\nabla \times \mathbf{E} = -\mu_0 \partial_t \mathbf{H}$$

$$\mathcal{L} = \begin{pmatrix} 0 & i\nabla\times \\ -i\nabla\times & 0 \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} \epsilon_0 n^2(\mathbf{r}) & 0 \\ 0 & \mu_0 \end{pmatrix}$$

$$\mathcal{F}_n^{(k)} = \begin{pmatrix} \mathbf{E}_n^{(k)} \\ \mathbf{H}_n^{(k)} \end{pmatrix}$$

$$\mathcal{L}\mathcal{F}_n^{(1)} - \omega_n \mathcal{M}\mathcal{F}_n^{(1)} = \mathcal{B}_n$$

$$\mathcal{B}_n = \begin{pmatrix} \imath\varepsilon_0 n^2(\mathbf{r}) \dfrac{da_n}{dt_1} \mathbf{E}_n^{(0)} + \imath \mathbf{J}_s^{(0)} \\ \imath\mu_0 \dfrac{da_n}{dt_1} \mathbf{H}_n^{(0)} \end{pmatrix}$$

$$t_n = \eta^n t$$

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Theory - Coupled Light Modes

If **PNL** is not zero, amplitudes depend on time and solution has the general form above
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Fredholm theorem: orthogonality with the kernel (0) solution:

$$\mathcal{F}_n : \qquad (\mathcal{F}_n, \mathcal{B}_n) = \int_V \mathcal{F}_n^* \cdot \mathcal{B}_n dV = 0$$

$$\mathbf{J}_n^{(0)} = -i\omega_n \mathbf{P}_n^{(0)}$$

yields

$$\frac{da_n(t)}{dt} = \imath \frac{\omega_n}{4} \int_V \mathbf{E}_n^*(\mathbf{r}) \cdot \mathbf{P}_n(\mathbf{r})\, dV$$

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Dynamics of Coupled Light Modes

Master equation

$$\frac{da_m(t)}{dt} = \frac{i\sqrt{\omega_m}}{4}\int_V \mathbf{E}^*_m(\mathbf{r})\cdot\mathbf{P}_m(\mathbf{r})d^3\mathbf{r}$$

$$\mathbf{P}_{NL}(\mathbf{r}) = \mathrm{Re}\left[\sum_m \sqrt{\omega_m}\, a_m(t)\mathbf{P}_m(\mathbf{r})e^{-i\omega_m t}\right]$$

$\alpha = x, y, z$
$m = 1, \ldots, N$

$$P^\alpha_m = \sum_{\omega_l} \chi^{(1)}_{\alpha\beta}(\omega_l, \omega_m; \mathbf{r})E^\beta_l(\mathbf{r})\sqrt{\omega_l}\, a_l + \sum_{\omega_j+\omega_k=\omega_l+\omega_m} \chi^{(3)}_{\alpha\beta\gamma\delta}(\omega_m; \omega_k, \omega_l, -\omega_j; \mathbf{r})E^\beta_j(\mathbf{r})E^\gamma_k(\mathbf{r})E^\delta_l(\mathbf{r})\sqrt{\omega_j\omega_k\omega_l}\, a^*_j a_k a_l$$

Optical response

Dynamics of Coupled Light Modes

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Optical response

Langevin equation

$$\frac{da_m(t)}{dt} = \sum_l \gamma_{lm} a_l + \sum_{[jkl]} g_{jklm} a^*_j a_k a_l + \Gamma_m(t)$$

$$\langle\Gamma_j(t)\Gamma^*_k(t')\rangle = 2k_B T\, \delta_{jk}\delta(t-t')$$

Spontaneous emission

Dynamics of Coupled Light Modes

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$$\frac{da_m(t)}{dt} = \frac{i\sqrt{\omega_m}}{4}\int_V \mathbf{E}_m^*(\mathbf{r})\cdot\mathbf{P}_m(\mathbf{r})d^3\mathbf{r}$$

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$$\omega_j - \omega_k = \omega_l - \omega_m$$

Mode-locking condition

$$\langle \Gamma_j(t)\Gamma_k^*(t')\rangle = 2k_BT\;\delta_{jk}\delta(t-t')$$

Spontaneous emission

$$\omega_j - \omega_k - \omega_l + \omega_m < \delta_\omega$$ spectral line width

Dynamics of Coupled Light Modes

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$$\frac{da_m(t)}{dt} = \frac{i\sqrt{\omega_m}}{4}\int_V \mathbf{E}^*_m(\mathbf{r})\cdot\mathbf{P}_m(\mathbf{r})d^3\mathbf{r}$$

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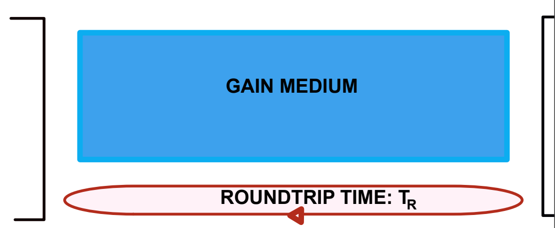
$$\langle \Gamma_j(t)\Gamma^*_k(t')\rangle = 2k_BT\;\delta_{jk}\delta(t-t')$$

Multimode mode-locking laser

- Once cavity is known, fields localization is known.
- Optical susceptibility can be computed in gas lasers (classical em field approx.) with two and three levels (and more..).
- To derive the optical susceptibility in solid state lasers is more complicated: em field must be quantizied. No known (by myself!) results for multimode lasing.



Dynamics of Coupled Light Modes

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Optical response

$$\frac{da_m(t)}{dt} = \sum_l \gamma_{lm} a_l + \sum_{[jkl]} g_{jklm} a^*_j a_k a_l + \Gamma_m(t)$$

Langevin equation

Spontaneous emission

$$\langle \Gamma_j(t)\Gamma^*_k(t')\rangle = 2k_B T\, \delta_{jk}\delta(t-t')$$

This generically holds for light modes interacting in a non-linearly polarized medium

What are the specific features of the modes of a Random Laser?

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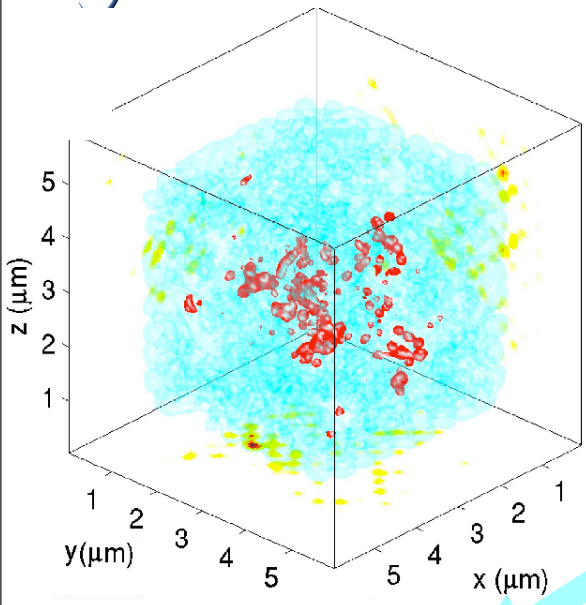
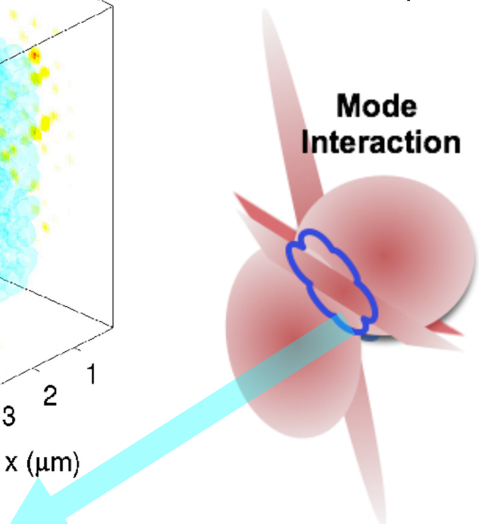
Stochastic Dynamics in Random Lasers



LOCALIZED MODES

S. Gentilini *et al.*, Opt. Lett **34**, 130 (2009).



RANDOMNESS

Cavity-less light amplification: modes are amplified by scattering through randomly placed dielectric particles. This implies **random spatial distribution of modes**, and induces **random susceptibility**.

Mode spatial overlaps modulated by non-linear susceptibility yield **quenched interactions** with (so far) **unknown probability distributions**

DISORDERED DISTRIBUTED INTERACTION COEFFICIENT

$$g_{jklm} = \imath \frac{\sqrt{\omega_j \omega_k \omega_l \omega_m}}{2} \int_V \chi^{(3)}_{\alpha,\beta.\gamma,\delta}(\omega_j,\omega_k,\omega_l,\omega_m) E^\alpha_m(\mathbf{r}) E^\beta_j(\mathbf{r}) E^\gamma_k(\mathbf{r}) E^\delta_l(\mathbf{r}) d^3\mathbf{r}$$

NONLINEAR RANDOM SUSCEPTIBILITY

α COMPONENT OF EM FIELD IN MODE m

$$\frac{da_m(t)}{dt} = \sum_l \gamma_{lm} a_l + \sum_{[jkl]} g_{jklm} a^*_j a_k a_l + \Gamma_m(t)$$

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NON-LINEARITY + RANDOMNESS

Stochastic Dynamics in Random Lasers

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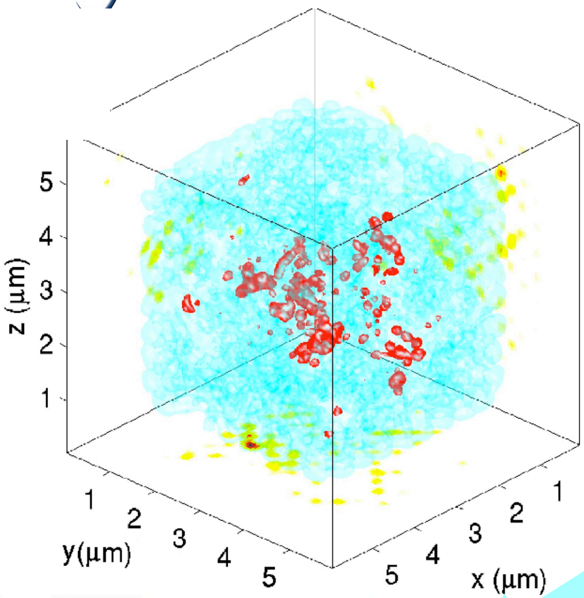

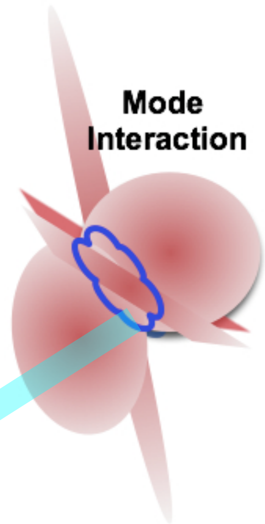

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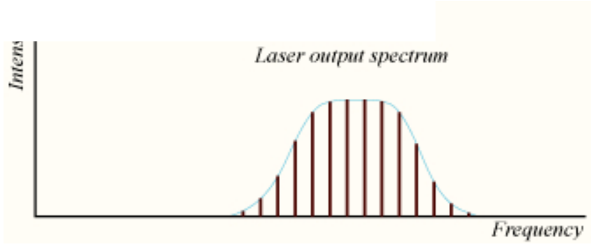
NON-LINEARITY + RANDOMNESS

Physical meaning of coefficients in master equation?

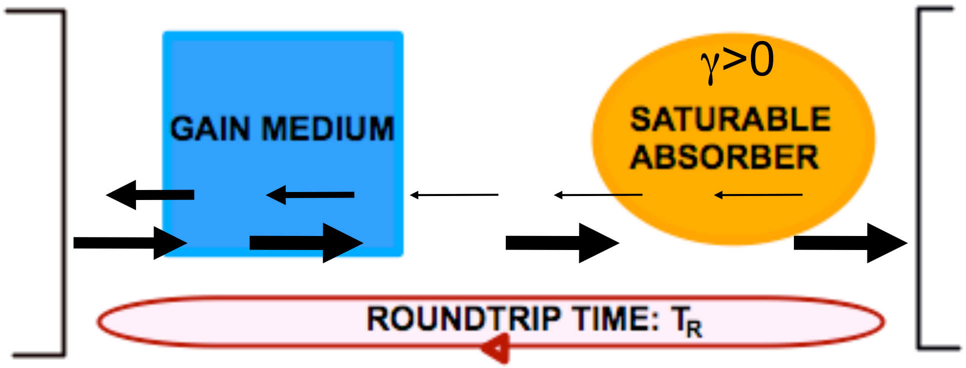
Standard Mode-Locking Lasers

$$\frac{da_m(t)}{dt} = \sum_{l} \gamma_{lm} a_l + \sum_{[jkl]} g_{jklm} a_j^* a_k a_l + \Gamma_m(t)$$

In absence of disorder the Master equation is the one of mode-locking lasers



$$\frac{\partial a_m}{\partial t} \equiv \dot{a}_m = [g_m - \ell_m + iD_m]a_m + (\gamma - i\delta) \sum_{\omega_j+\omega_k-\omega_l=\omega_m} a_j^\star a_k a_l + \Gamma_m(t)$$



HA Haus, *Waves and Fields in Optoelectonics, 1984*

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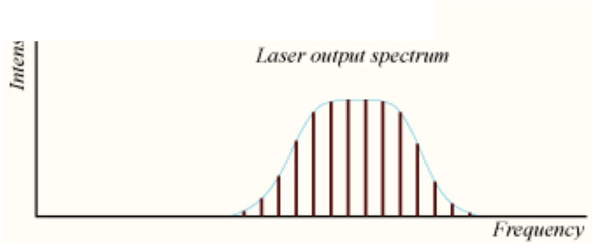

GAIN LOSS GROUP VELOCITY DISPERSION SATURABLE ABSORBER KERR LENS EFFECT SPONTANEOUS EMISSION

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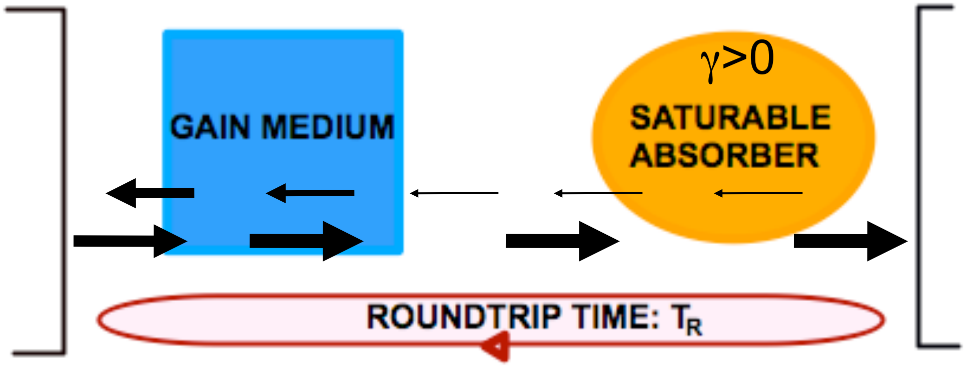

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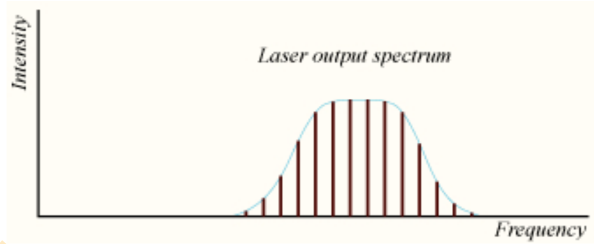

GAIN — LOSS — GROUP VELOCITY DISPERSION — SATURABLE ABSORBER — KERR LENS EFFECT — SPONTANEOUS EMISSION

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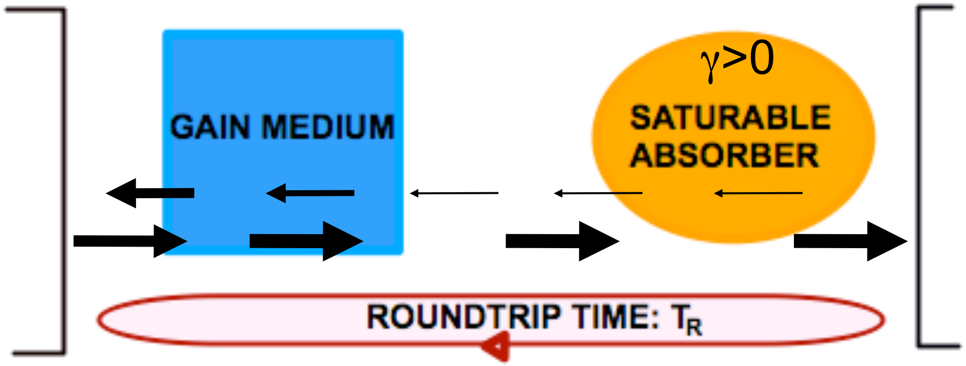

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$$\gamma \sim \text{Re}\,[g]$$

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Statistical Mechanics of waves in nonlinear disordered media

From Langevin to Hamilton

- $$\frac{da_m(t)}{dt} = \sum_l \gamma_{lm} a_l + \sum_{[jkl]} g_{jklm} a_j^* a_k a_l + \Gamma_m(t) \longrightarrow \dot{a}_j = -\frac{\partial \mathcal{H}}{\partial a_j^*} + \Gamma_j$$

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Statistical Mechanics of Random Lasers

From Langevin to Hamilton

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From instantaneous EM energy of the non-linear (closed) cavity to Hamiltonian

- $$\mathcal{E} = \frac{1}{2}\int \left[\mathbf{D}(\mathbf{r})\cdot\mathbf{E}(\mathbf{r}) + \mu_0|\mathbf{H}(\mathbf{r})|^2\right] dV = \sum_m \mathcal{N}_m \omega_m |a_m(t)|^2 + \frac{1}{2}\int \mathbf{E}(\mathbf{r})\cdot\mathbf{P}_{NL}(\mathbf{r}) dV$$

$$\longrightarrow \mathcal{H} = -\frac{1}{2}\langle \int \mathbf{E}(\mathbf{r})\cdot\mathbf{P}_{NL}(\mathbf{r}) dV \rangle_{\text{optical cycle}}$$

Statistical Mechanics of Random Lasers

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Hamiltonian description:

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk} \gamma_{jk} a_j a_k^* + \sum_{[jklm]} g_{jklm} a_j a_k^* a_l a_m^*\right]$$

Localized mode interaction mediated by radiating modes	Non-linear localized mode-coupling expressed by 4-body interaction between amplitudes

with a global energy constraint

$$\mathcal{E} = \sum_k \omega_k |a_k|^2$$

Mean-Field model with Gaussian distribution of couplings:
2+4 spherical spin-glass [A Crisanti & LL, PRL 04, PRB 06, PRB 07, NPB 13]

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Topology? Light modes Network?

Hamiltonian description:

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk} \gamma_{jk} a_j a_k^* + \sum_{[jklm]} g_{jklm} a_j a_k^* a_l a_m^*\right]$$

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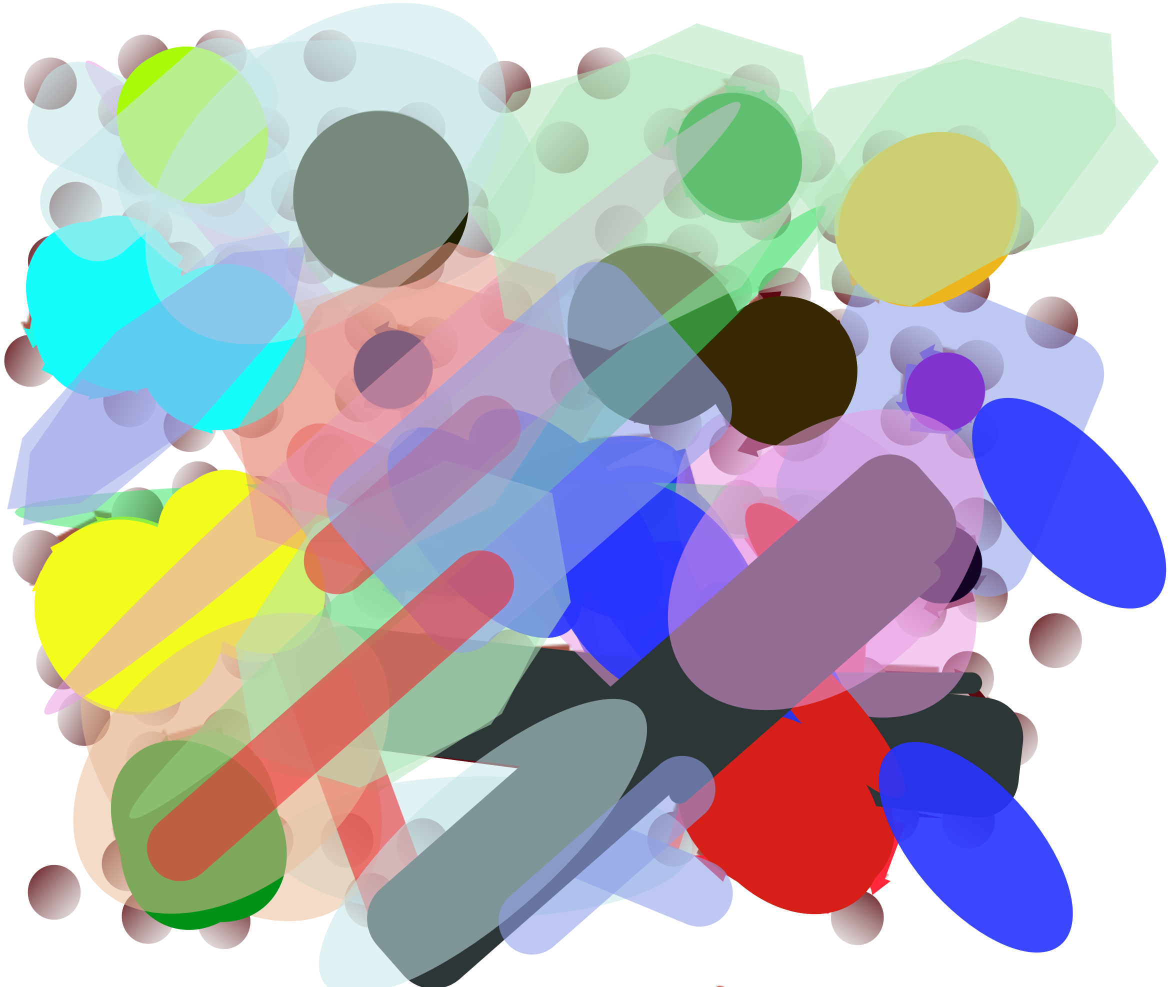
Modeling random laser modes in space:
the electromagnetic field of each localized light mode of frequency ω is non-zero in a given region of space **r**.

Modes interaction depends on their spatial overlap (4-uple).



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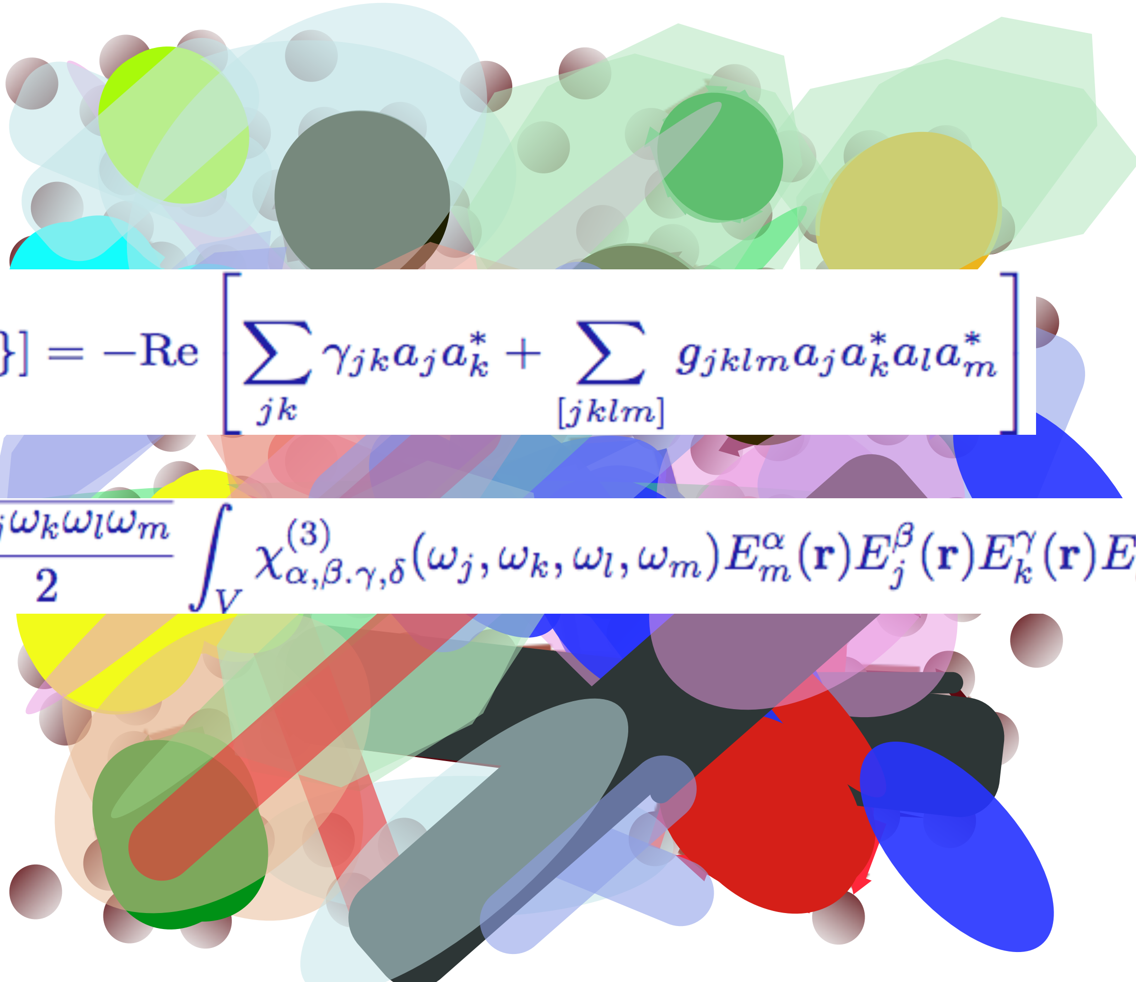

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk} \gamma_{jk} a_j a_k^* + \sum_{[jklm]} g_{jklm} a_j a_k^* a_l a_m^*\right]$$

$$g_{jklm} = \imath \frac{\sqrt{\omega_j \omega_k \omega_l \omega_m}}{2} \int_V \chi^{(3)}_{\alpha,\beta.\gamma,\delta}(\omega_j, \omega_k, \omega_l, \omega_m) E_m^\alpha(\mathbf{r}) E_j^\beta(\mathbf{r}) E_k^\gamma(\mathbf{r}) E_l^\delta(\mathbf{r}) d^3\mathbf{r}$$

DIPARTIMENTO di FISICA

NETADIS
Statistical Physics Approaches to Networks Across Disciplines

SAPIENZA UNIVERSITÀ DI ROMA

Modeling random laser modes in space:
the electromagnetic field of each localized light mode of frequency ω is non-zero in a given region of space **r**.

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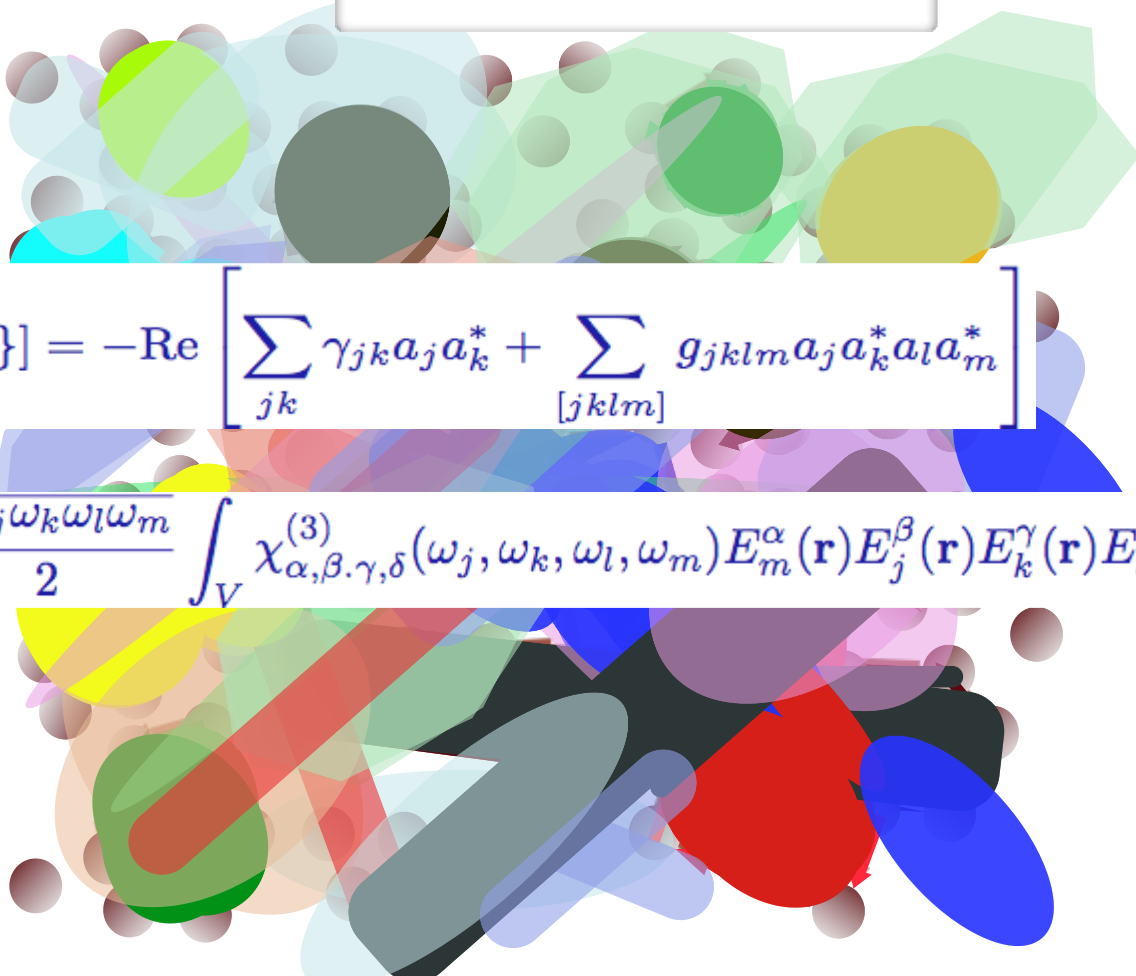
$$\omega_j - \omega_k = \omega_l - \omega_m$$



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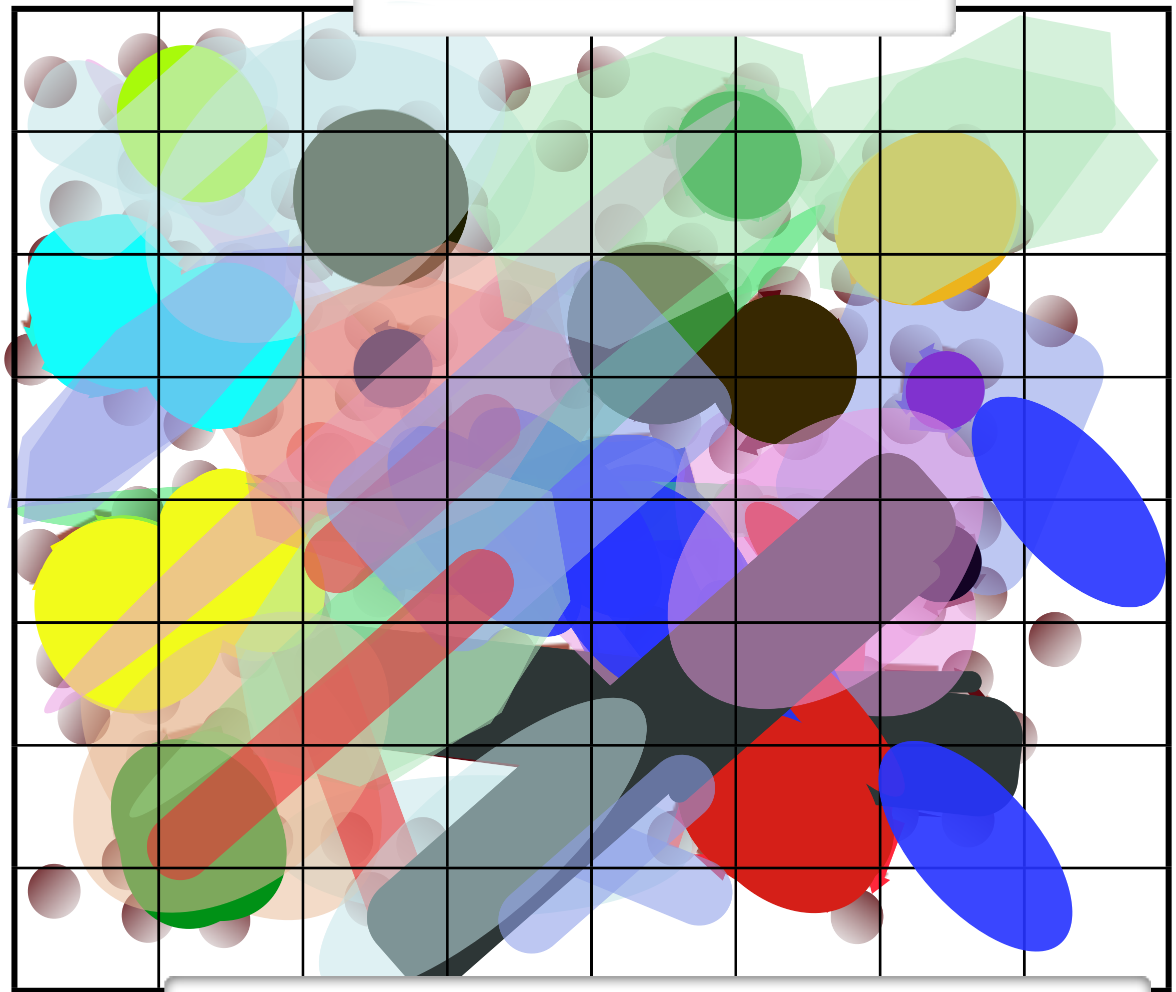
Modeling random laser modes in space:
localized modes can be **coarse-grained** as nodes of a graph with links to other nodes if their spatial overlap is non-zero and the ... satisfied.

$$\omega_j - \omega_k = \omega_l - \omega_m$$

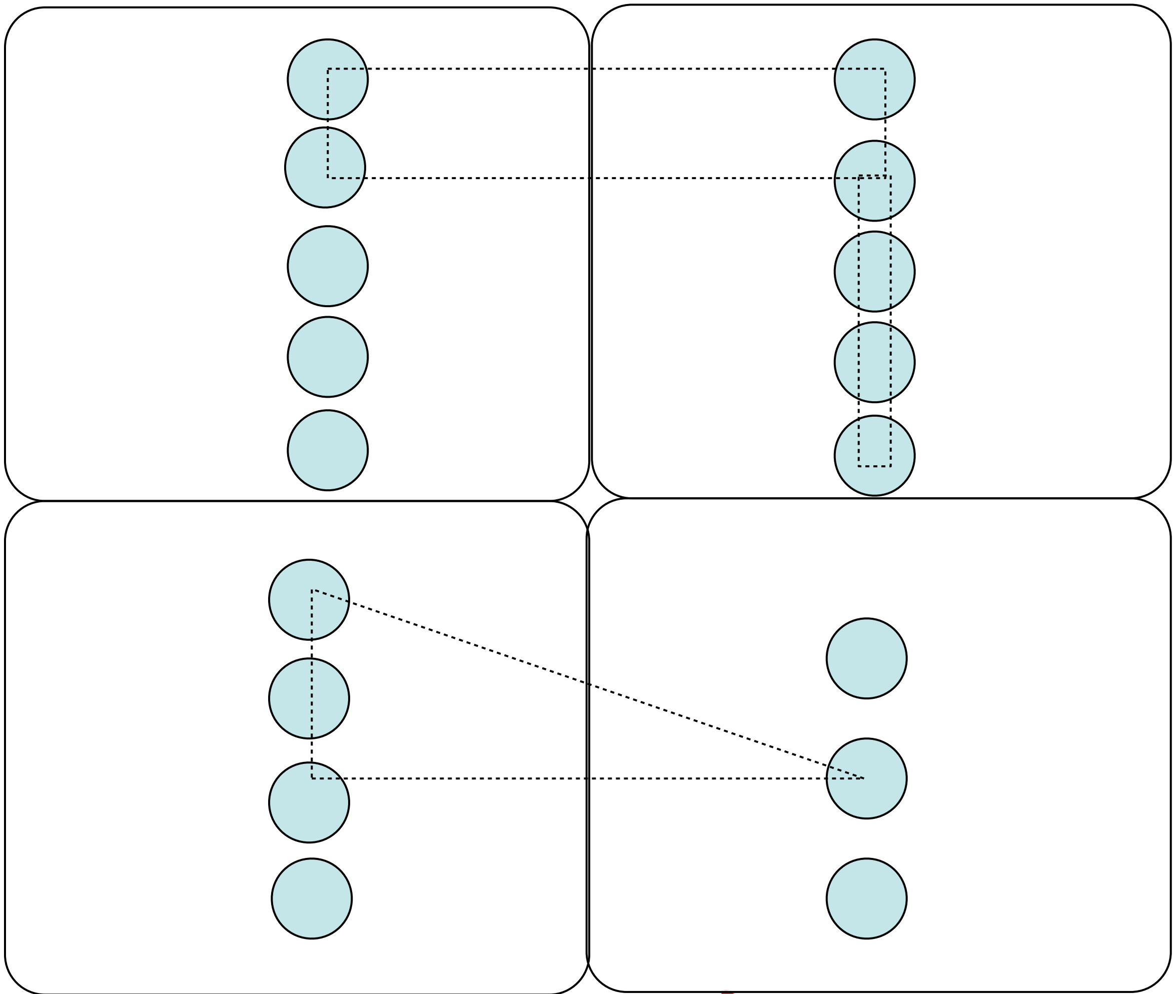
$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk} \gamma_{jk} a_j a_k^* + \sum_{[jklm]} g_{jklm} a_j a_k^* a_l a_m^*\right]$$



$$g_{jklm} \propto \int_V \chi^{(3)}_{\alpha,\beta.\gamma,\delta}(\omega_j,\omega_k,\omega_l,\omega_m) E^{\alpha}_m(\mathbf{r}) E^{\beta}_j(\mathbf{r}) E^{\gamma}_k(\mathbf{r}) E^{\delta}_l(\mathbf{r}) d^3\mathbf{r}$$

Modeling random laser modes in space: localized modes can be coarse-grained as nodes of a graph with links to other nodes if their spatial overlap is non-zero and the mode-locking condition is satisfied.

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk} \gamma_{jk} a_j a_k^* + \sum_{[jklm]} g_{jklm} a_j a_k^* a_l a_m^*\right]$$



M modes in each cell

DIPARTIMENTO di FISICA

NETADIS Statistical Physics Approaches to Networks Across Disciplines

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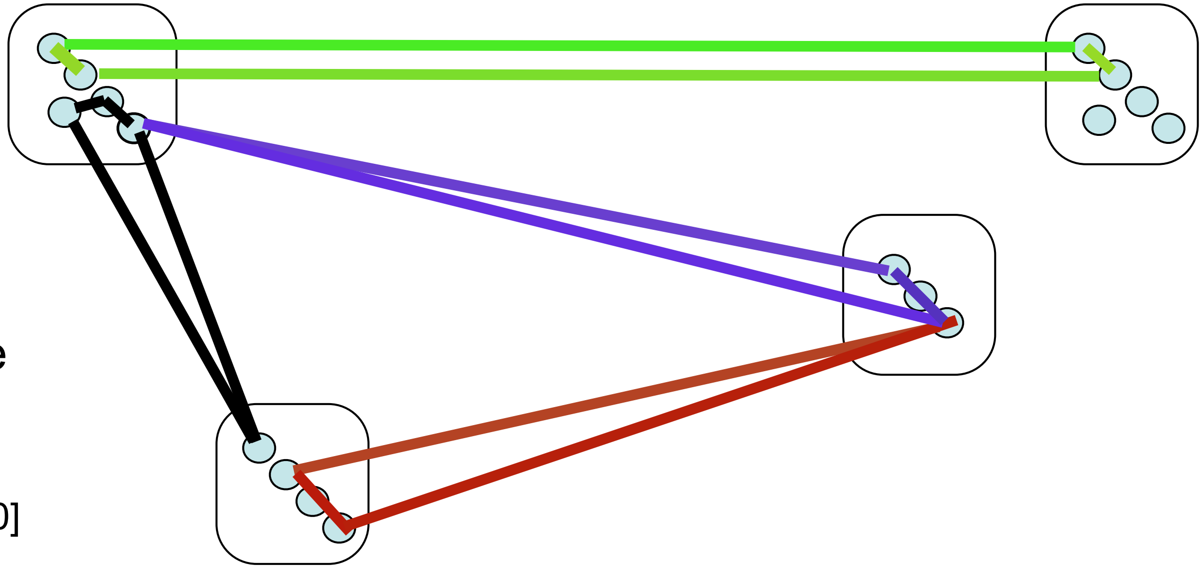
Modeling random laser modes in space:
localized modes can be coarse-grained as nodes of a graph with links to other nodes if their spatial overlap is non-zero and the mode-locking condition is satisfied.

M modes in each cell.
A cell is a node in a network.

4-mode but 2-node interaction
~(M,p) model
[Caltagirone et al. PRB10]



Once all non-zero “4-modes” couplings have been selected: network of cells/nodes, each one containing M modes.

Selection “tools”:

$$\omega_j - \omega_k = \omega_l - \omega_m$$

+

$$g_{jklm} \propto \int_V \chi^{(3)}_{\alpha,\beta.\gamma,\delta}(\omega_j,\omega_k,\omega_l,\omega_m) E^{\alpha}_m(\mathbf{r}) E^{\beta}_j(\mathbf{r}) E^{\gamma}_k(\mathbf{r}) E^{\delta}_l(\mathbf{r}) d^3\mathbf{r}$$

Summing up and moving forward

Hamiltonian description:

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk}\gamma_{jk}a_j a_k^* + \sum_{[jklm]} g_{jklm}a_j a_k^* a_l a_m^*\right]$$

Localized mode interaction mediated by radiating modes	Non-linear localized mode-coupling expressed by 4-body interaction between amplitudes

with a global energy constraint

$$\mathcal{E} = \sum_k |a_k|^2$$

Statistical Mechanics allows to model different kinds of random lasers characterized by:

• ***degree of disorder*** (ranging from almost standard lasers with an irreducible noise to completely random lasers),

• ***extension of modes*** localization,
• ***geometry and dimension***, ***interaction range***

• ***pumping*** intensity and mode-locked pulse length,

• role of **radiating modes** in modulating localized modes linear/'two-body' interaction (assumption of pure self-interaction),

• characteristic ***times of magnitude and phase*** of complex mode amplitudes (assumption of quenched amplitudes)

Statistical Mechanics of Random Lasers

Hamiltonian description:

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk} \gamma_{jk} a_j a_k^* + \sum_{[jklm]} g_{jklm} a_j a_k^* a_l a_m^*\right]$$

Localized mode interaction mediated by radiating modes	Non-linear localized mode-coupling expressed by 4-body interaction between amplitudes

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Statistical Mechanics allows to model different kinds of random lasers characterized by:

- ***degree of disorder*** $\longrightarrow$ **COUPLING DISTRIBUTION PARAMETER VALUES (mean, variance)**
- ***extension of modes*** localization,
- ***geometry and dim.***, ***interaction range*** } **NETWORK/GRAPH STRUCTURE**
- ***pumping*** intensity $\longrightarrow$ **TEMPERATURE**
- role of **radiating modes** in modulating localized modes 'two-body' interaction $\longrightarrow$ $\sum_{jk} \gamma_{jk} a_j a_k^* \simeq \sum_j \gamma_j |a_j|^2$
- characteristic ***times of magnitude and phase*** of complex mode amplitudes $\longrightarrow$ $a_j = A_j \, e^{\imath \phi_j}$

SLOW / **FAST**

Random laser Hamiltonian zoology

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk}\gamma_{jk}a_j a_k^* + \sum_{[jklm]} g_{jklm}a_j a_k^* a_l a_m^*\right]$$

$$\mathcal{E} = \sum_k |a_k|^2$$

$$\omega_j - \omega_k = \omega_m - \omega_l$$

$$\mathcal{H}[\{a_j\}] \simeq -\sum_j \gamma_j |a_j|^2 - \mathrm{Re}\left[\sum_{[jklm]} g_{jklm}a_j a_k^* a_l a_m^*\right]$$

Negligible non-local interaction between localized modes via radiation modes (closed cavity)

$$\mathcal{H}[\{A_j;\phi_j\}] \simeq -\sum_j \gamma_j A_i^2 - \sum_{[jklm]} A_j A_k A_l A_m \mathrm{Re}\left[g_{jklm}e^{\imath(\phi_i-\phi_k+\phi_l-\phi_m)}\right]$$

Decoupled amplitude magnitude and phase

Random laser Hamiltonian zoology II

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk} \gamma_{jk} a_j a_k^* + \sum_{[jklm]} g_{jklm} a_j a_k^* a_l a_m^*\right]$$

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Quenched approximation for amplitudes

$$\mathcal{H}[\{\phi_j\}] \simeq -\sum_{[j,k,l,m]} J_{jklm} \cos\left(\phi_j - \phi_k + \phi_l - \phi_m\right)$$

$$J_{jklm} \propto A_j A_k A_l A_m g_{jklm}$$

Couplings and Localization

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk}\gamma_{jk}a_j a_k^* + \sum_{[jklm]} g_{jklm} a_j a_k^* a_l a_m^*\right]$$

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Inferring g's yields information about ***light modes localization***

$$g_{jklm} \propto \int_V \chi^{(3)}_{\alpha,\beta.\gamma,\delta}(\omega_j,\omega_k,\omega_l,\omega_m) E_m^\alpha(\mathbf{r}) E_j^\beta(\mathbf{r}) E_k^\gamma(\mathbf{r}) E_l^\delta(\mathbf{r}) d^3\mathbf{r}$$

mode coupling
in position (nodes)
and frequency

$$j \begin{cases} \omega_j \\ \mathbf{r}_j \end{cases}$$

Can we measure g's?

Inverse problem in waves

$$\mathcal{H}[\{a_j\}] = -\mathrm{Re}\left[\sum_{jk}\gamma_{jk}a_j a_k^* + \sum_{[jklm]} g_{jklm}a_j a_k^* a_l a_m^*\right] \qquad \mathcal{E} = \sum_k |a_k|^2$$

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Inferring g's yields information about light modes localization

$$C^{(4)}_{jklm} = \langle\cos(\phi_j - \phi_k + \phi_l - \phi_m)\rangle \qquad j\left\{\begin{array}{l}\omega_j\\ \mathbf{r}_j\end{array}\right.$$

$$C^{(4)}_{jklm} = \langle\mathrm{Re}\,[a_j a_k^* a_l\, a_m^*]\rangle \qquad C^{(2)}_{jklm} = \langle\mathrm{Re}\,[a_j a_k^*]\rangle$$

Inverse problem in waves

$$g_{jklm} \propto \int_V \chi^{(3)}_{\alpha,\beta.\gamma,\delta}(\omega_j,\omega_k,\omega_l,\omega_m) E^\alpha_m(\mathbf{r}) E^\beta_j(\mathbf{r}) E^\gamma_k(\mathbf{r}) E^\delta_l(\mathbf{r}) d^3\mathbf{r}$$

Inferring g's yields information about light modes localization

Fitting graphical problem techniques can be applied/generalized, though:

Theory

- variables are continuous: XY (phases) or spherical (amplitudes) "spins";

- quenched disorder is there;

- four point correlations have to be considered, besides two point.

$$C^{(4)}_{jklm} = \langle \cos(\phi_j - \phi_k + \phi_l - \phi_m) \rangle$$

$$C^{(4)}_{jklm} = \langle \mathrm{Re}\ [a_j a^*_k a_l\, a^*_m] \rangle \qquad C^{(2)}_{jklm} = \langle \mathrm{Re}\ [a_j a^*_k] \rangle$$

Experiments

- Intensities (mode magnitudes) are available versus frequency.
How many modes? From 10 to 10^5, though refinement is finite in spectra (e.g., .3 nm) -> 100:1000 distinct frequencies can be usually appreciated;

- phases (vs. frequency) have not been measured yet: set up is ready, probing standard lasers and seeking high intensity random lasers (rhodamine+TiO_2 is not "energetic" enough).

Inverse problem in waves

$$g_{jklm} \propto \int_V \chi^{(3)}_{\alpha,\beta.\gamma,\delta}(\omega_j,\omega_k,\omega_l,\omega_m) E^\alpha_m(\mathbf{r}) E^\beta_j(\mathbf{r}) E^\gamma_k(\mathbf{r}) E^\delta_l(\mathbf{r}) d^3\mathbf{r}$$

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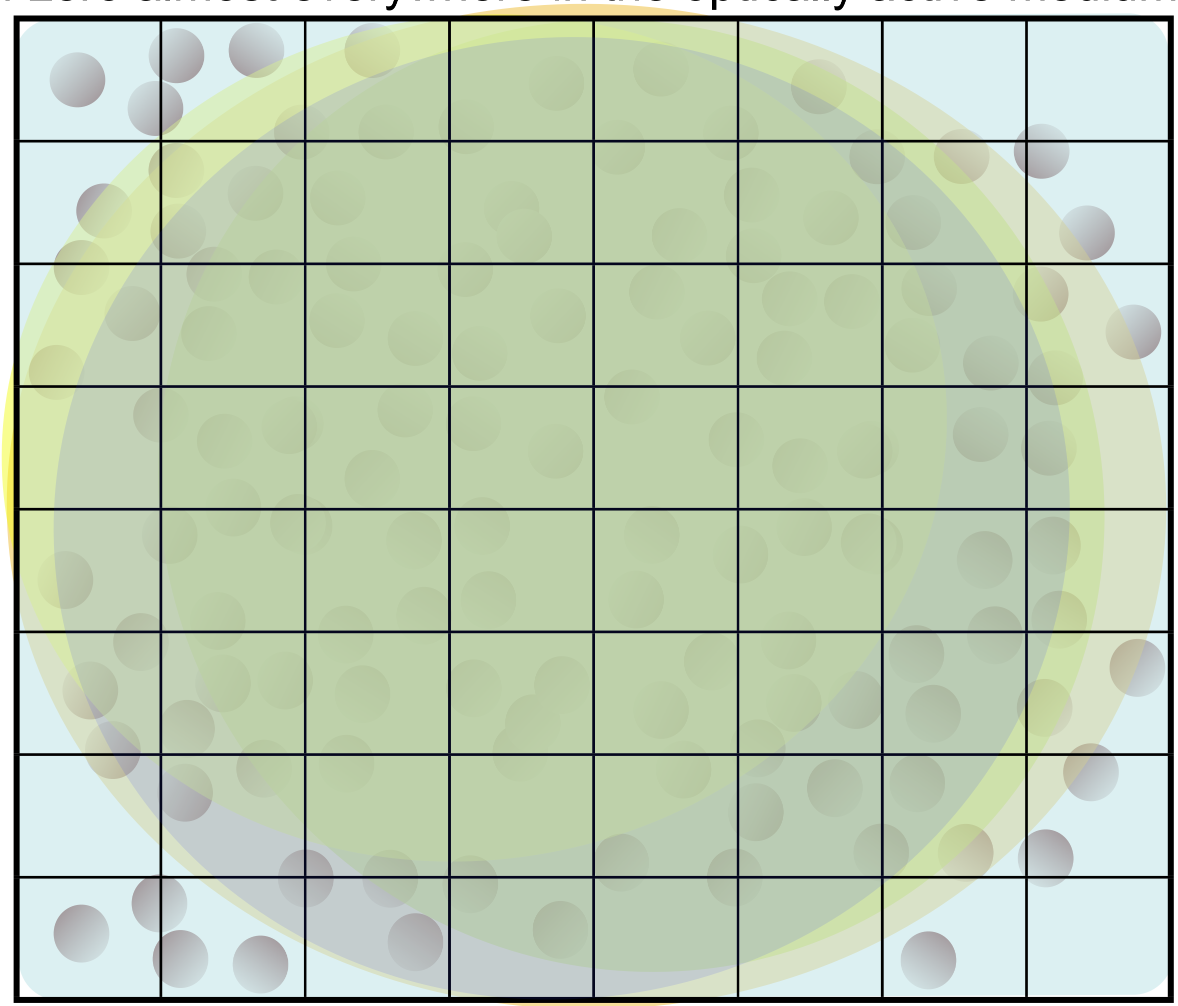
"Warm up" with:

- ordered multimode mode-locking laser;
- linearly interacting waves (not lasers): $C^{(2)}_{jklm} = \langle \text{Re } [a_j a^*_k] \rangle$

Mean-field model for slow amplitudes

In some systems modes can be localized
but non-zero almost everywhere in the optically active medium



$$\mathcal{H}[\{A_j;\phi_j\}] \simeq -\sum_j \gamma_j A_i^2 - \sum_{[jklm]} A_j A_k A_l A_m \mathrm{Re}\left[g_{jklm} e^{\imath(\phi_i-\phi_k+\phi_l-\phi_m)}\right]$$

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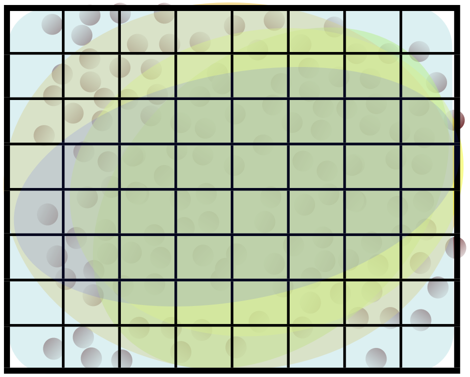

Mode-locking condition

$$\omega_j - \omega_k = \omega_m - \omega_l$$

implies dilution:
~ Erdos-Renyi random graph [Tyagi]

Phase model

$$\mathcal{H}[\{A_j;\phi_j\}] \simeq -\sum_j \gamma_j A_i^2 - \sum_{[jklm]} A_j A_k A_l A_m \mathrm{Re}\left[g_{jklm} e^{\imath(\phi_i - \phi_k + \phi_l - \phi_m)}\right]$$

$$\mathcal{H}[\{A_j;\phi_j\}] \simeq -\mathcal{P}^2 \sum_{[jklm]} \mathrm{Re}\left[J_{jklm} e^{\imath(\phi_j - \phi_k + \phi_l - \phi_m)}\right];$$

$$\mathcal{P} \propto \langle A^2 \rangle$$

$$J_{jklm} \equiv g_{jklm} A_j A_k A_l A_m / \mathcal{P}^2$$

Mean-field model for slow amplitudes

In some systems modes can be localized
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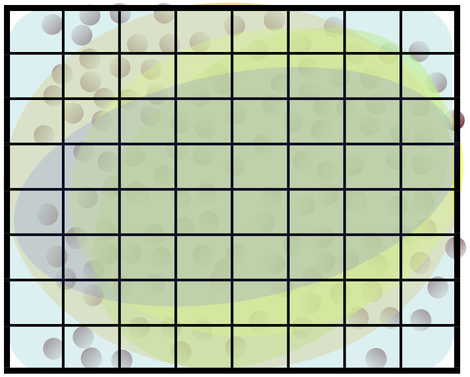


Mode-locking condition

$$\omega_j - \omega_k = \omega_m - \omega_l$$

implies dilution: Erdos-Renyi/ Bethe lattice [Tyagi]

$$\mathcal{H}[\{A_j;\phi_j\}] \simeq -\sum_j \gamma_j A_i^2 - \sum_{[jklm]} A_j A_k A_l A_m \mathrm{Re}\left[g_{jklm} e^{\imath(\phi_i-\phi_k+\phi_l-\phi_m)}\right]$$

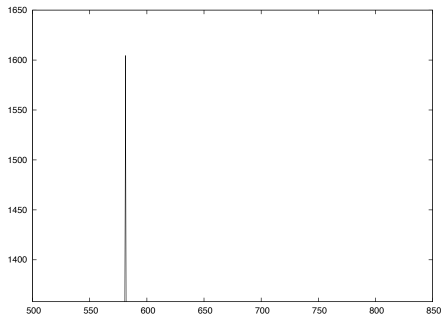
$$\mathcal{H}[\{A_j;\phi_j\}] \simeq -\mathcal{P}^2 \sum_{[jklm]} \mathrm{Re}\left[J_{jklm} e^{\imath(\phi_j-\phi_k+\phi_l-\phi_m)}\right]; \qquad \mathcal{P} \propto \langle A^2 \rangle, \quad J_{jklm} \equiv g_{jklm} A_j A_k A_l A_m / \mathcal{P}^2$$

Fully connected Mean-field approximation

Mode-locking always satisfied

$$\omega_j - \omega_k = \omega_m - \omega_l$$

←



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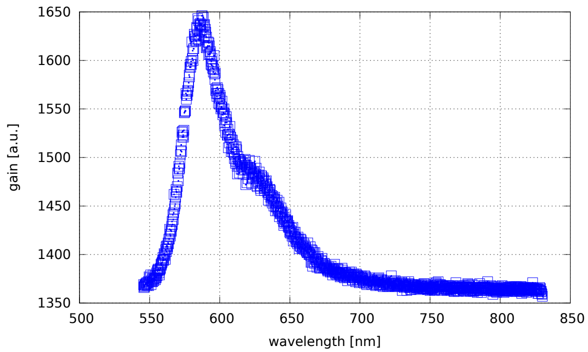

$$\mathcal{H}[\{\phi_j\}] \simeq -\mathcal{P}^2 \sum_{j<k;l<m;k<l} J_{jklm} \cos\left(\phi_j - \phi_k + \phi_l - \phi_m\right)$$

Mean-field model for slow amplitudes

$$\mathcal{H}[\{\phi_j\}] \simeq -\mathcal{P}^2 \sum_{j<k;l<m;k<l} J_{jklm} \cos\left(\phi_j - \phi_k + \phi_l - \phi_m\right)$$

Mean-field approximation:

- all 4-plets of phases interact with each other with small couplings (vanishing in the thermodynamic limit $N \to \infty$) - GEOMETRY
- bandwidth narrows and the spectral distribution of angular frequencies is peaked around a value: $\omega_j \sim \omega_0$ for all modes $j=1,\ldots,N$ - MODE LOCKING

condition $\omega_j - \omega_k = \omega_l - \omega_m$ is always satisfied

Gaussian independent identically distributed interaction couplings

$$\langle J_{jklm} \rangle = J_0/N^3$$
$$\langle (J_{jklm} - \langle J_{jklm} \rangle)^2 \rangle = \sigma_J/N^3$$

$$\mathcal{P}^2 = J_0 \frac{\langle A^2 \rangle^2}{k_B T_{\text{bath}}}$$

PARTITION FUNCTION:

$$Z_J = \int \prod_{j=1}^{N} d\phi_j \; e^{-\beta \mathcal{P}^2 \mathcal{H}[\{\phi_k\}]} = \int \prod_{j=1}^{N} d\phi_j \; e^{-\tilde{\beta} \mathcal{H}[\{\phi_k\}]}; \qquad \tilde{\beta} = \mathcal{P}^2 \beta$$

Statistical mechanical properties, thermodynamic phases, order parameters,.....

Mean-field replica theory

Mean-field approximation

$$\mathcal{H}[\{\phi_j\}] \simeq - \sum_{i<j;l<m;i<l} J_{jklm} \cos\left(\phi_i - \phi_k + \phi_l - \phi_m\right)$$

$$\langle J_{jklm} \rangle = J_0/N^3$$
$$\langle (J_{jklm} - \langle J_{jklm} \rangle)^2 \rangle = \sigma_J/N^3$$

Gaussian independent identically distributed interaction couplings

the role of inverse temperature is played by the square of the average stored energy per mode: "pumping rate"

$$Z_J = \int \prod_{j=1}^{N} d\phi_j \; e^{-\beta \mathcal{P}^2 \mathcal{H}[\{\phi_k\}]} = \int \prod_{j=1}^{N} d\phi_j \; e^{-\tilde{\beta}\mathcal{H}[\{\phi_k\}]}; \qquad \tilde{\beta} = \mathcal{P}^2 \beta$$

$$\tilde{\beta} = \mathcal{P}^2/k_B T$$
$$\mathcal{P} \propto \langle \mathcal{E} \rangle = \langle |a_j|^2 \rangle$$
$$R_J \equiv \sigma_J/J_0$$

?

$$\beta\Phi = -\frac{1}{N}\langle \log Z_J \rangle_J = -\frac{1}{N}\lim_{n\to 0}\frac{\langle Z_J^n \rangle - 1}{n}$$

Replica trick

In mean-field replica calculation sites interaction is eliminated and replicas interaction is introduced through the ***overlap*** order parameters

$$q_{ab} = \langle e^{\imath(\phi_a - \phi_b)} \rangle \qquad r_{ab} = \langle e^{\imath(\phi_a + \phi_b)} \rangle$$

plus standard o.p.'s ("*magnetizations*")

$$m_a = \langle e^{\imath\phi_a} \rangle \qquad \tilde{r}_a = \langle e^{2\imath\phi_a} \rangle$$

Mean-field replica theory

$$\beta\Phi = -\frac{1}{N}\langle \log Z_J \rangle_J = -\frac{1}{N}\lim_{n\to 0}\frac{\langle Z_J^n \rangle - 1}{n}$$

$$\langle Z \rangle \simeq \int \mathcal{D}\phi \int \prod_{jklm} dJ_{jklm} e^{-J_{jklm}^2} e^{\mathcal{H}[\phi]} \simeq \int \mathcal{D}\phi \int \prod_{jklm} dJ_{jklm} e^{-J_{jklm}^2} e^{J_{jklm}\cos(\ldots\phi\ldots)}$$

$$\langle Z^n \rangle \simeq \prod_{a=1}^n \left[\int \mathcal{D}\phi^{(a)} \int \prod_{jklm} dJ_{jklm} e^{-J_{jklm}^2} e^{\mathcal{H}[\phi^{(a)}]} \right]$$
$$\simeq \int \prod_{a=1}^n \mathcal{D}\phi^{(a)} \int \prod_{jklm} dJ_{jklm} e^{-J_{jklm}^2} e^{J_{jklm}\sum_{a=1}^n \cos(\ldots\phi^{(a)}\ldots)}$$

All replicas enter in the same way: symmetry

Solving the thermodynamics replica symmetry is *spontaneously* broken: RSB theory

NETADIS
Statistical Physics Approaches to Networks Across Disciplines

Statistical Mechanics of Random Lasers

Mean-field approximation

$$\mathcal{H}[\{\phi_j\}] \simeq - \sum_{i<j;l<m;i<l} J_{jklm} \cos(\phi_i - \phi_k + \phi_l - \phi_m)$$

$$\langle J_{jklm}\rangle = J_0/N^3$$
$$\langle (J_{jklm} - \langle J_{jklm}\rangle)^2\rangle = \sigma_J/N^3$$

$$\tilde{\beta} = \mathcal{P}^2/k_B T$$
$$\mathcal{P} \propto \langle \mathcal{E}\rangle = \langle |a_j|^2\rangle$$
$$R_J \equiv \sigma_J/J_0$$

$$\beta\Phi = -\frac{1}{N}\langle \log Z_J\rangle_J = -\frac{1}{N}\lim_{n\to 0}\frac{\langle Z_J^n\rangle - 1}{n}$$

Replica trick

It is “correct”, e.g., thermodynamically stable / self-consistent
in the “**one-step Replica Symmetry Breaking**” scheme of computation

$$P(X) = m\,\delta(X - X_0) + (1-m)\delta(X - X_1)$$

G.Parisi, 1979, 1980

“REPLICATED” FREE ENERGY

$$\bar{\beta}\Phi = -\frac{\bar{\beta}R_J}{8}\overset{1}{|\tilde{m}|^4} - \frac{\bar{\beta}^2}{32}\left[1 - (1 - \overset{2}{m})\left(|\overset{3}{q_1}|^4 + |\overset{4}{r_1}|^4\right) - m\left(|\overset{5}{q_0}|^4 + |\overset{6}{r_0}|^4\right) + |\overset{7}{r_d}|^2\right]$$
$$-\mathrm{Re}\left[\frac{1-m}{2}\left(\overset{8}{\bar{\lambda}_1} q_1 + \overset{9}{\bar{\mu}_1} r_1\right) + \frac{m}{2}\left(\overset{10}{\bar{\lambda}_0} q_0 + \overset{11}{\bar{\mu}_0} r_0\right) - \overset{12}{\bar{\mu}_d} r_d - \overset{13}{\bar{\nu}}\tilde{m}\right]$$
$$+\frac{\lambda_1^R}{2} - \frac{1}{m}\int \mathcal{D}[0]\log\int\mathcal{D}[1]\left[\int_0^{2\pi} \mathbf{d}\phi\ \exp\mathcal{L}(\phi;\mathbf{0},\mathbf{1})\right]^{\mathbf{m}}$$

Statistical Mechanics of Random Lasers

$$\mathcal{H}[\{\phi_j\}] \simeq -\sum_{i<j;l<m;i<l} J_{jklm}\cos(\phi_i-\phi_k+\phi_l-\phi_m)$$

$$\langle J_{jklm}\rangle = J_0/N^3$$
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Mean-field approximation

Gaussian independent identically distributed interaction couplings

$$\tilde\beta = \mathcal{P}^2/k_BT$$
$$\mathcal{P}\propto\langle\mathcal{E}\rangle = \langle|a_j|^2\rangle$$
$$R_J\equiv\sigma_J/J_0$$

"REPLICATED" FREE ENERGY

$$\bar\beta\Phi = -\frac{\bar\beta R_J}{8}|\tilde m|^4 - \frac{\bar\beta^2}{32}\left[1-(1-m)\left(|q_1|^4+|r_1|^4\right)-m\left(|q_0|^4+|r_0|^4\right)+|r_d|^2\right]$$
$$-\mathrm{Re}\left[\frac{1-m}{2}\left(\bar\lambda_1 q_1+\bar\mu_1 r_1\right)+\frac{m}{2}\left(\bar\lambda_0 q_0+\bar\mu_0 r_0\right)-\bar\mu_d r_d-\bar\nu\tilde m\right]$$
$$+\frac{\lambda_1^R}{2}-\frac{1}{m}\int\mathcal{D}[\mathbf{0}]\log\int\mathcal{D}[\mathbf{1}]\left[\int_0^{2\pi}\mathbf{d}\phi\ \exp\mathcal{L}(\phi;\mathbf{0},\mathbf{1})\right]^{\mathbf{m}}$$

LL et al PRL 09
C Conti and LL, PRB 11

$$\mathcal{L}(\phi;\mathbf{0},\mathbf{1})\equiv\mathrm{Re}\left\{e^{i\phi}\left[\bar\zeta_1\sqrt{\Delta\lambda^R-|\Delta\mu|}+\bar\zeta_0\sqrt{\lambda_0^R-|\mu_0|}+x_1\sqrt{2\Delta\bar\mu}+x_0\sqrt{2\bar\mu_0}+\bar\nu\right]+e^{2i\phi}\left(\bar\mu_d-\frac{\bar\mu_1}{2}\right)\right\}$$

$$\Delta\lambda=\lambda_1-\lambda_0\qquad\Delta\mu=\mu_1-\mu_0\qquad\mathbf{0}=\{x_0,\zeta_0^R,\zeta_0^I\}\qquad\mathbf{1}=\{x_1,\zeta_1^R,\zeta_1^I\}$$

$$\mathcal{D}[\mathbf{a}]=\frac{1}{(2\pi)^{3/2}}\exp\left[-\frac{(x_a^2+(\zeta_a^R)^2+(\zeta_a^I)^2)}{2}\right]$$

SELF-CONSISTENCY EQUATIONS

$$\lambda_{0,1}=\frac{\bar\beta^2}{4}(q_{0,1})^3\ ;\quad \mu_{0,1}=\frac{\bar\beta^2}{4}|r_{0,1}|^2\,r_{0,1}$$
$$\mu_d=\frac{\bar\beta^2}{8}|r_d|^2 r_d\ ;\quad \nu=\frac{\bar\beta R_J}{2}|\tilde m|^2\tilde m$$
$$q_1=\langle\langle c_{\mathcal{L}}^2\rangle_m\rangle_0+\langle\langle s_{\mathcal{L}}^2\rangle_m\rangle_0$$
$$q_0=\langle\langle c_{\mathcal{L}}\rangle_m^2\rangle_0+\langle\langle s_{\mathcal{L}}\rangle_m^2\rangle_0$$
$$r_1=\langle\langle c_{\mathcal{L}}^2\rangle_m\rangle_0-\langle\langle s_{\mathcal{L}}^2\rangle_m\rangle_0+2i\langle\langle c_{\mathcal{L}}s_{\mathcal{L}}\rangle_m\rangle_0$$
$$r_0=\langle\langle c_{\mathcal{L}}\rangle_m^2\rangle_0-\langle\langle s_{\mathcal{L}}\rangle_m^2\rangle_0+2i\langle\langle c_{\mathcal{L}}\rangle_m\rangle_0\langle\langle s_{\mathcal{L}}\rangle_m\rangle_0$$
$$r_d=\langle\langle\langle e^{2i\phi}\rangle_{\mathcal{L}}\rangle_m\rangle_0;\qquad \tilde m=\langle\langle\langle e^{i\phi}\rangle_{\mathcal{L}}\rangle_m\rangle_0$$

$$\langle(\ldots)\rangle_m\equiv\frac{\int\mathcal{D}[\mathbf{1}](\ldots)\left[\int_0^{2\pi}d\phi\ e^{\mathcal{L}(\phi;\mathbf{0},\mathbf{1})}\right]^m}{\int\mathcal{D}[\mathbf{1}]\left[\int_0^{2\pi}d\phi\ e^{\mathcal{L}(\phi;\mathbf{0},\mathbf{1})}\right]^m}$$

$$\langle(\ldots)\rangle_0\equiv\int\mathcal{D}[\mathbf{0}](\ldots)$$

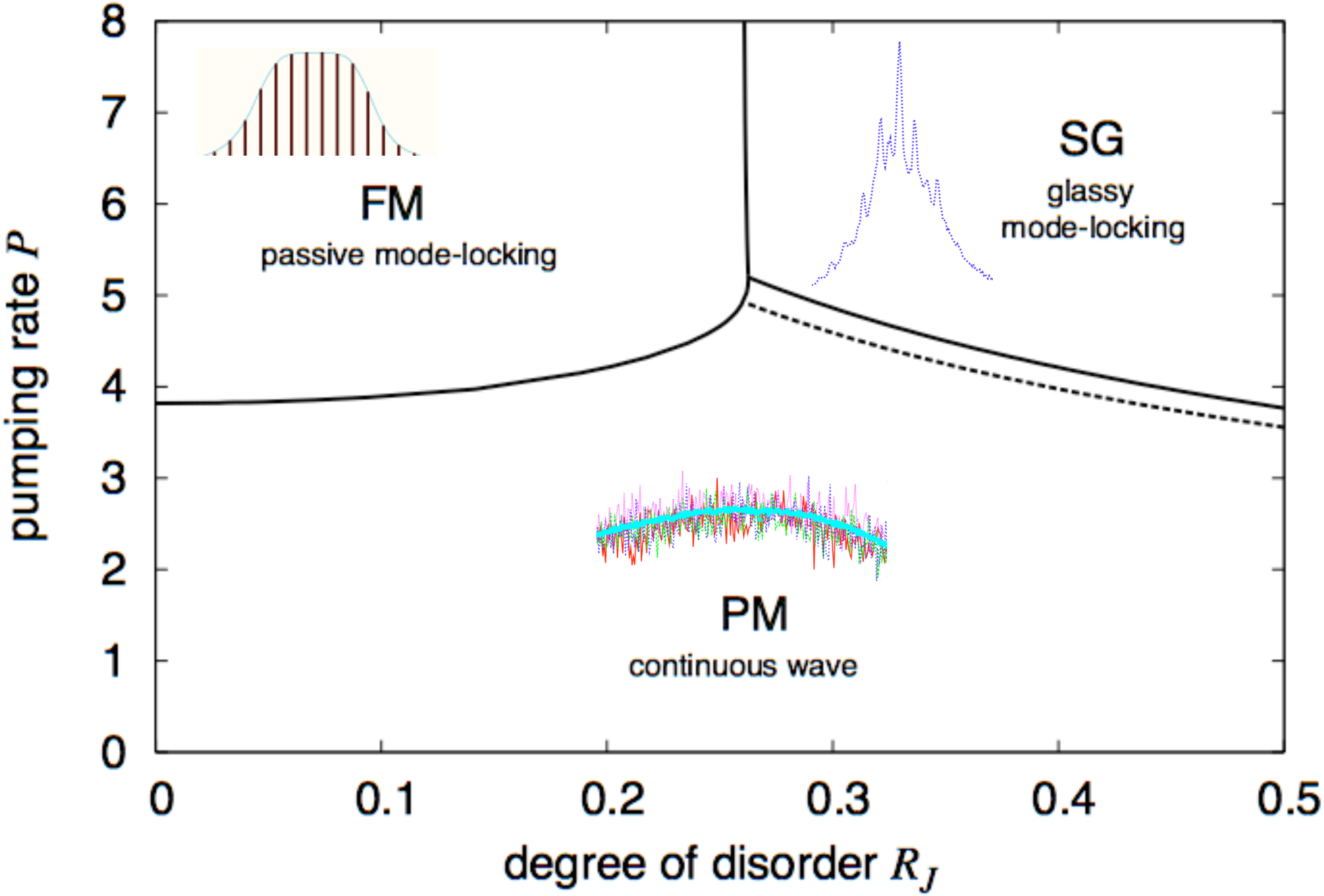
N.B.:
RSB parameter
m still
undetermined

Photonic phase diagram

Mean-field approximation

$$\mathcal{H}[\{\phi_j\}] \simeq - \sum_{i<j;l<m;i<l} J_{jklm} \cos(\phi_i - \phi_k + \phi_l - \phi_m)$$

Photonics: pumping vs disorder



Phase diagram

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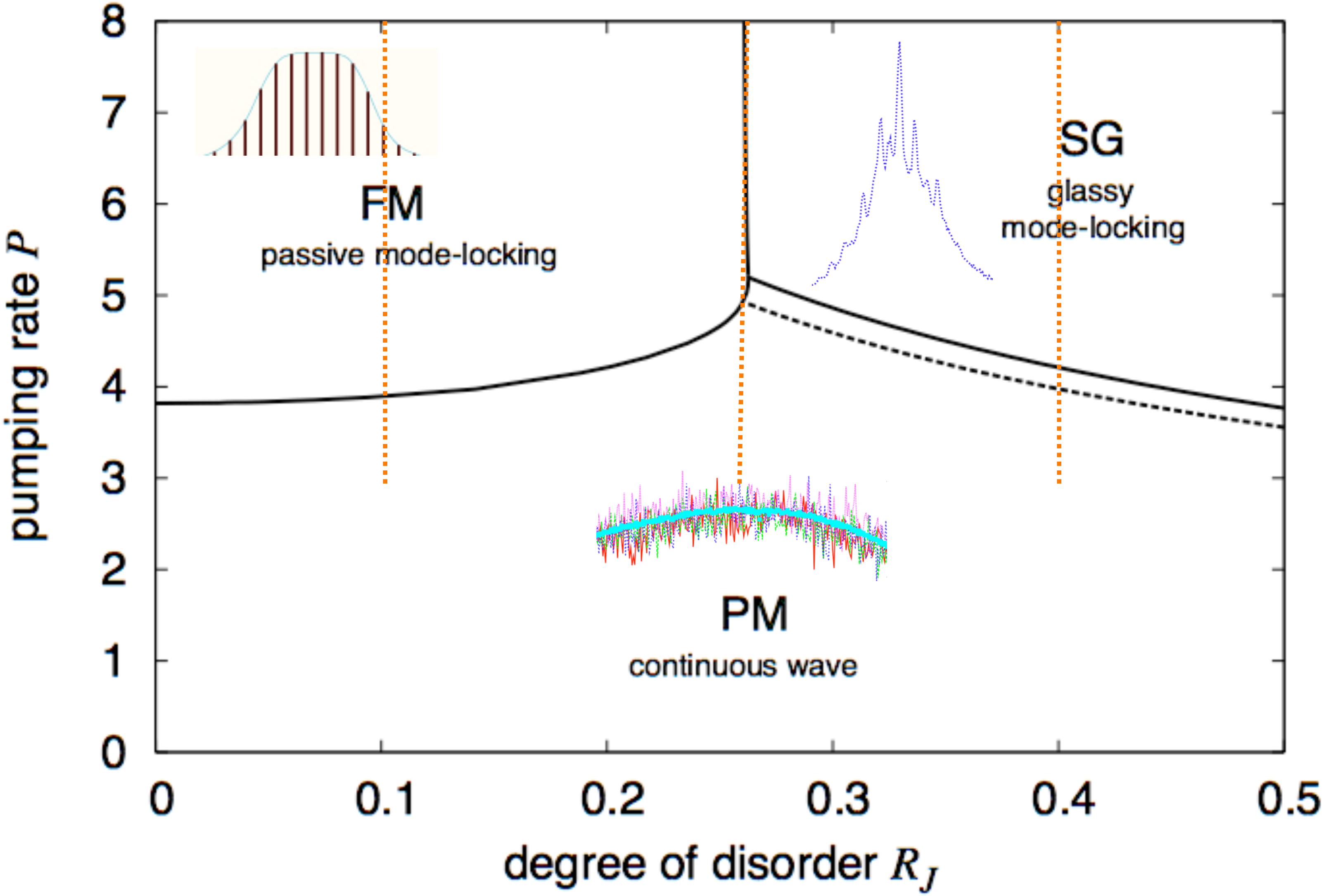
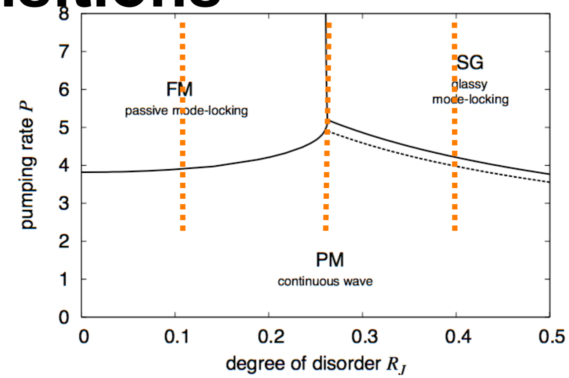


Across transitions



Mean-field approximation

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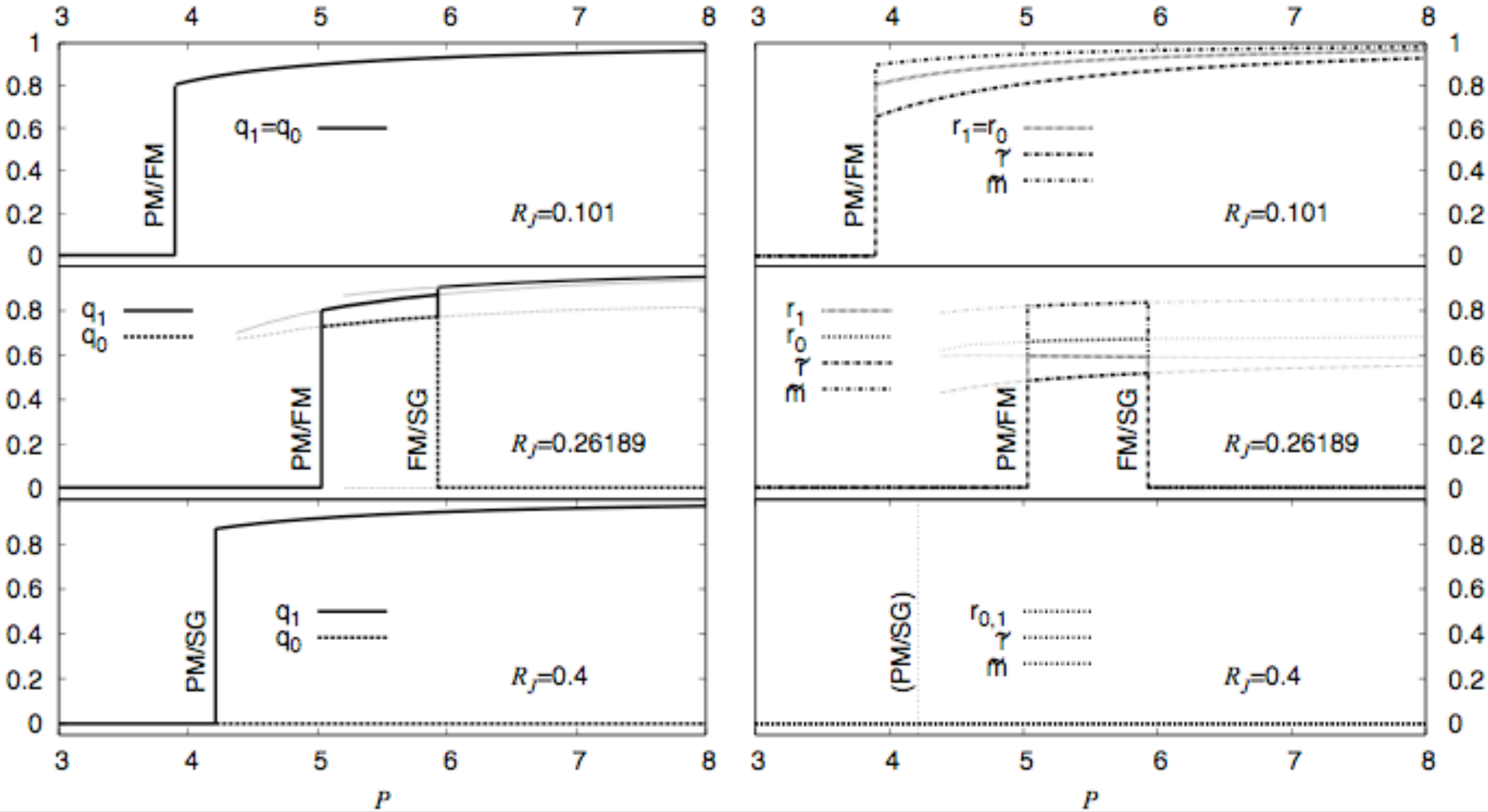
Discontinuous order parameters



Outlook and Perspectives

So far....

* Experimental spectral evidence compatible with the conjecture that **random laser** thermodynamics/dynamics is ruled by **complex free energy landscape**.

* **Physical replicas** realization: in experiments the quenched disorder can be kept constant for different measurements.

* RL behavior can be modeled by **Hamiltonian models with quenched disorder** and effects of tuning disorder strength can be predicted.
[control of disorder is fundamental in the physics and engineering of **nano-structured lasers** (and “cavity-less” lasers)].

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In progress....

* “photonic spin-glasses” can model and reproduce: (i) **fluorescence/random laser transition** [Angelani et al. PRL06], (ii) **ordered/random laser transition** [LL et al PRL09, Conti & LL PRB11] (e.g., granulars [Folli et al. PRL12], nano crystal lasers,..) (iii) **random laser spectra**.

* Inference of non-linear couplings yield information about **modes localization** and optical response in random (and non-random) lasers. Graphical problems techniques.

* Construction of **quantitative models** (real distribution of disorder, total energy profile with pumping, diluted interactions, finite dimensional structure, …).

* Applications to other wave problems in nonlinear random media (BEC in temperature, optical propagation at T=0) [Conti & LL PRB11].

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* Experimental spectral evidence compatible with the conjecture that **random laser** thermodynamics/dynamics is ruled by **complex free energy landscape**.
* **Physical replicas** realization: in experiments the quenched disorder can be kept constant for different measurements [Leonetti].
* RL behavior can be modeled by **Hamiltonian models with quenched disorder** and effects of tuning disorder strength can be predicted [LL et al PRL09].

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In progress....

* "photonic spin-glasses" can model and reproduce: (i) **fluorescence/random laser transition** , (ii) **ordered/random laser transition** [LL et al PRL09, Conti & LL PRB11] (e.g., in granulars Folli et al. PRL12) (iii) **random laser spectra** [Antenucci, Tyagi].
* Inference of non-linear couplings yield information about **modes localization** and optical response in random (and non-random) lasers. Graphical problems techniques [Tyagi].
* Non-linear susceptibility computation for multimode gas and solid state lasers, small disorder effects [Marruzzo].
* Construction of **quantitative models** (real distribution of disorder, total energy profile with pumping, diluted interactions, finite dimensional structure, …) [Antenucci, Ibanez, Tyagi].
* Applications to other wave problems in nonlinear random media (BEC in temperature, optical propagation at T=0) [Conti & LL PRB11].

Experimentally...

* Experiments to measure **phases** rather than intensities [Ghofraniha] and directly access the dynamic variables would allow for correlation measure and theory test and inference.
* Experiments in which the **degree of disorder is tuned changing the compactness of granular stochastic resonators** in random lasing materials [Folli et al. PRL12].