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Contagion Across Credit Networks

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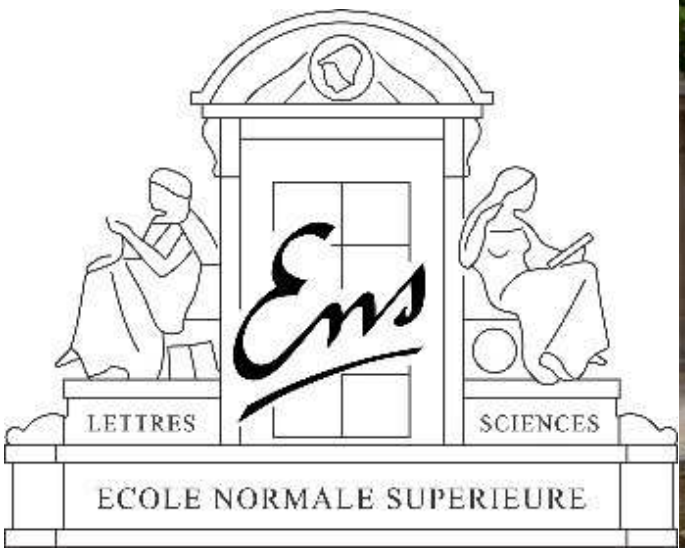
Personal Background

- BSc, MSc at the École Normale Superieure de Paris, France



- MSc in quantum mechanics, specialized in statistical physics
- previous research projects in granular media, climate science and out-of-equilibrium statistical physics

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Project background

- Hatchett & Kühn 2006 : *Effect of economic interactions on credit risk* presents a simple model of economic interaction and study the contagion effects.
- The goal was to present a qualitative picture of the effects of interactions on firms' default risk, showing that a firm's default risk is highly dependent on its environment.
- Unlike many regulatory models, it is stochastic in nature

Project background : Simple model

- Given a collection of N nodes, we draw according to specified distribution
  - a collection of edges e_{ij}
  - an interaction variable J_{ij} on each edge
  - an initial wealth position ϑ_i on each node
  - a status variable $n_{i,t}$ on each node.A node where $n_{i,t} = 1$ is considered to have defaulted
  - a collection $\eta_{i,t} = \eta_0 + \xi_{i,t}$ of "noise" random variables on each node
- The system is then propagated along $T = 12$ time steps according to the dynamics

$$n_{i,t+1} = n_{i,t} + (1 - n_{i,t})\Theta\Big(\sum_j J_{ij} n_{j,t} - \vartheta_i + \eta_{i,t}\Big)$$

Project background : Simple model

In the large connectivity limit, using gaussian interactions and noise, the system evolves according to an effective dynamics

$$n_{t+1} = n_t + \frac{1-n_t}{2}\left[1 + \mathrm{erf}\left(\frac{\bar{J}m_t + \sigma^2_{\eta_0} - \vartheta}{\sqrt{2(\sigma^2_\xi + \sigma^2_J m_t)}}\right)\right]$$

$$m_{t+1} = m_t + \left\langle \frac{1-n_t}{2}\left[1 + \mathrm{erf}\left(\frac{\bar{J}m_t + \sigma^2_{\eta_0} - \vartheta}{\sqrt{2(\sigma^2_\xi + \sigma^2_J m_t)}}\right)\right]\right\rangle_{(\vartheta)}$$

where m_t is the average defaulted fraction.

Project background : credit derivatives

- After the 2007-2008 crisis highlighted the role of financial derivatives, the model was adapted to take into account credit derivatives such as Credit Default Swaps (CDS) (Heise and Kühn 2012, *Derivatives and Credit Contagion in Interconnected Networks*)
- The model incorporates different sectors and types of interactions, including three-body interactions
- What happens when more realistic parameters are used (scale-free networks, liquidity shocks...) ?

In the new model, three types of new interactions are introduced :

- unhedged lending : A bank lends a certain amount of money to a firm, and receives interest as a result as long as the firm hasn't defaulted.

$$L_{i,t}^{(u)} = \sum_{j} J_{ij}^{(u)} \left[n_{j,\tau} - \sum_{\tau=1}^{t} \epsilon_{ij,\tau} \right]$$

- Hedged loans : A bank (insurance buyer) lends to a firm (reference entity), then insures the loan with another bank (insurance seller) for a refular fee. If the reference entity defaults while the insurance seller hasn't defaulted, the loss is taken by the insurance seller.

$$L_{i,t}^{(hb)} = \sum_{j,k} J_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{k,\tau} - n_{k,\tau-1}) n_{j,\tau} + f_{ij,\tau}^{k} - \epsilon_{ij\tau} \right]$$

$$L_{i,t}^{(hs)} = - \sum_{j,k} J_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{k,\tau} - n_{k,\tau-1})(1 - n_{j,\tau}) - f_{ij,\tau}^{k} \right]$$

- speculative insurance, where a protection buyer insures against the default of an entity it hasn't contracted a loan with

$$L_{i,t}^{(sb)} = -\sum_{j,k} K_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{k,\tau - n_{k,\tau-1}})(1 - n_{j,\tau}) - f_{ij,\tau}^{k} \right]$$

$$L_{i,t}^{(ss)} = \sum_{j,k} K_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{k,\tau - n_{k,\tau-1}})(1 - n_{j,\tau}) - f_{ij,\tau}^{k} \right]$$

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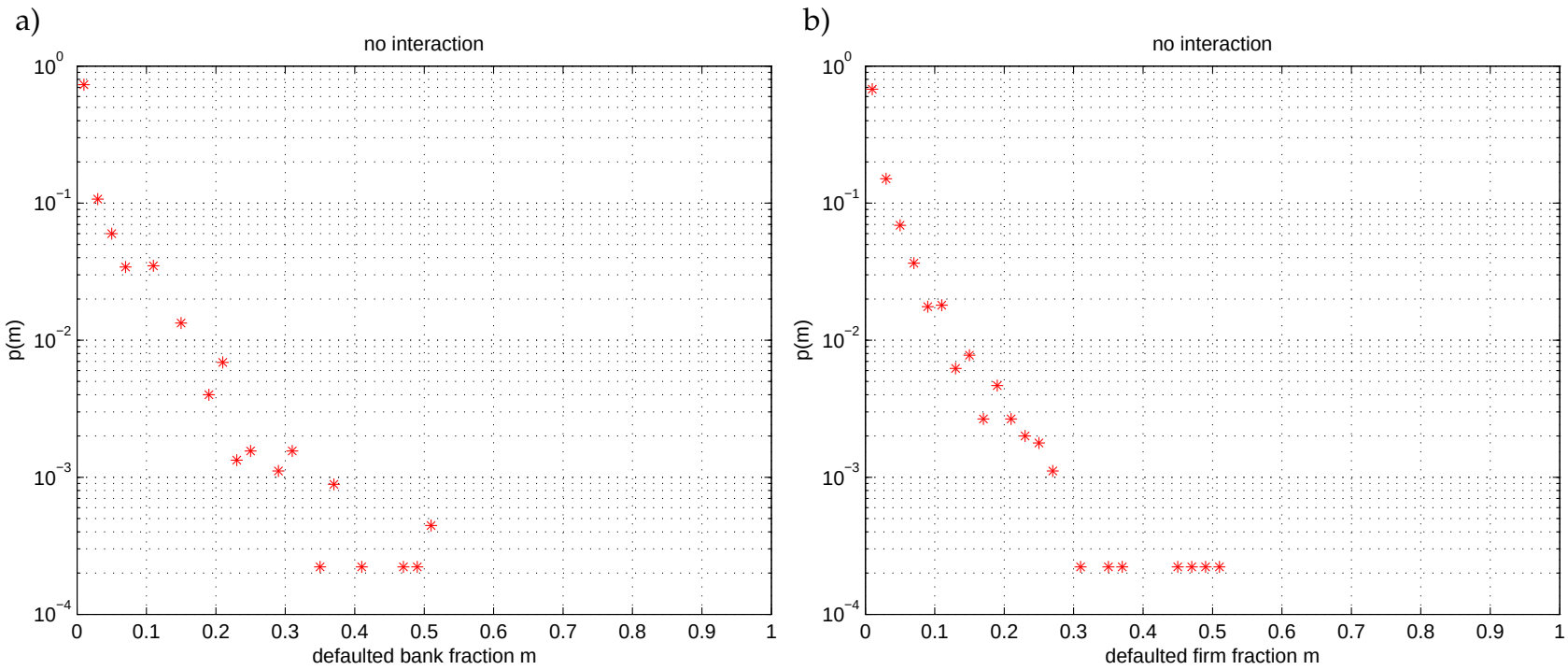
The project so far

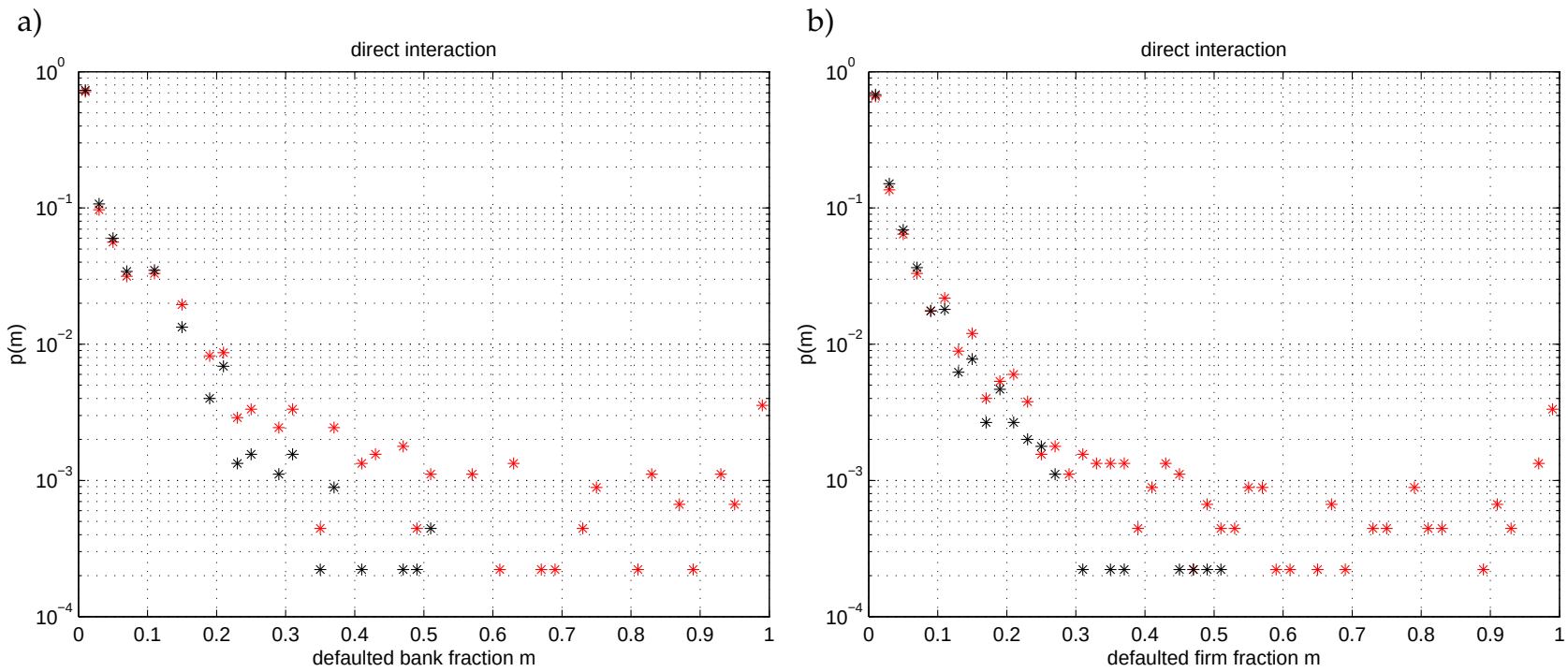
- I have written a program to do numerical simulations of both simple and CDS model, which agree well with analytical predictions, and can be used to include further correlations
- We have moved toward a different parametrisation of the interaction, emphasizing the leverage ration instead of monetary value.
- analytic solutions are in progress

What are we looking at ?



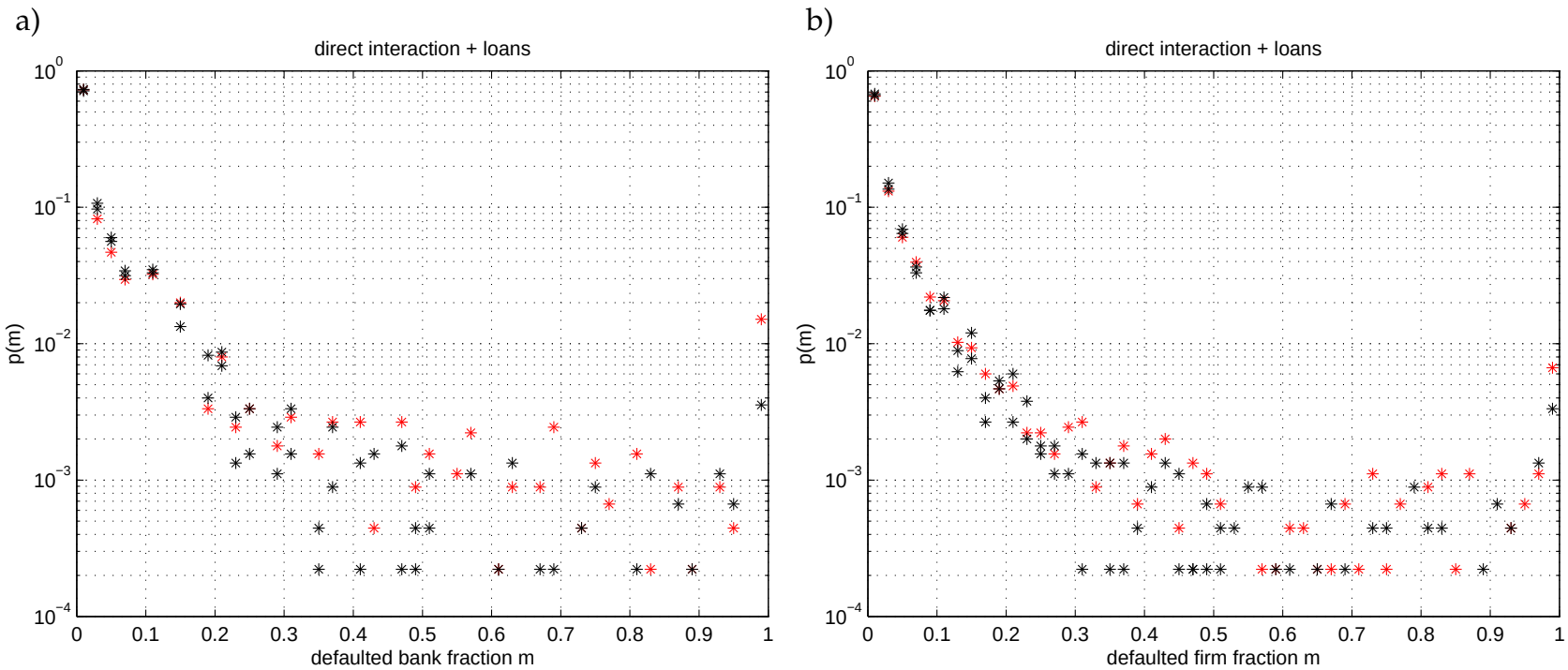
distribution of default fractions over 4500 runs in the absence of interactions :
a) in the banking sector
b) in the firms sector

What are we looking at ?



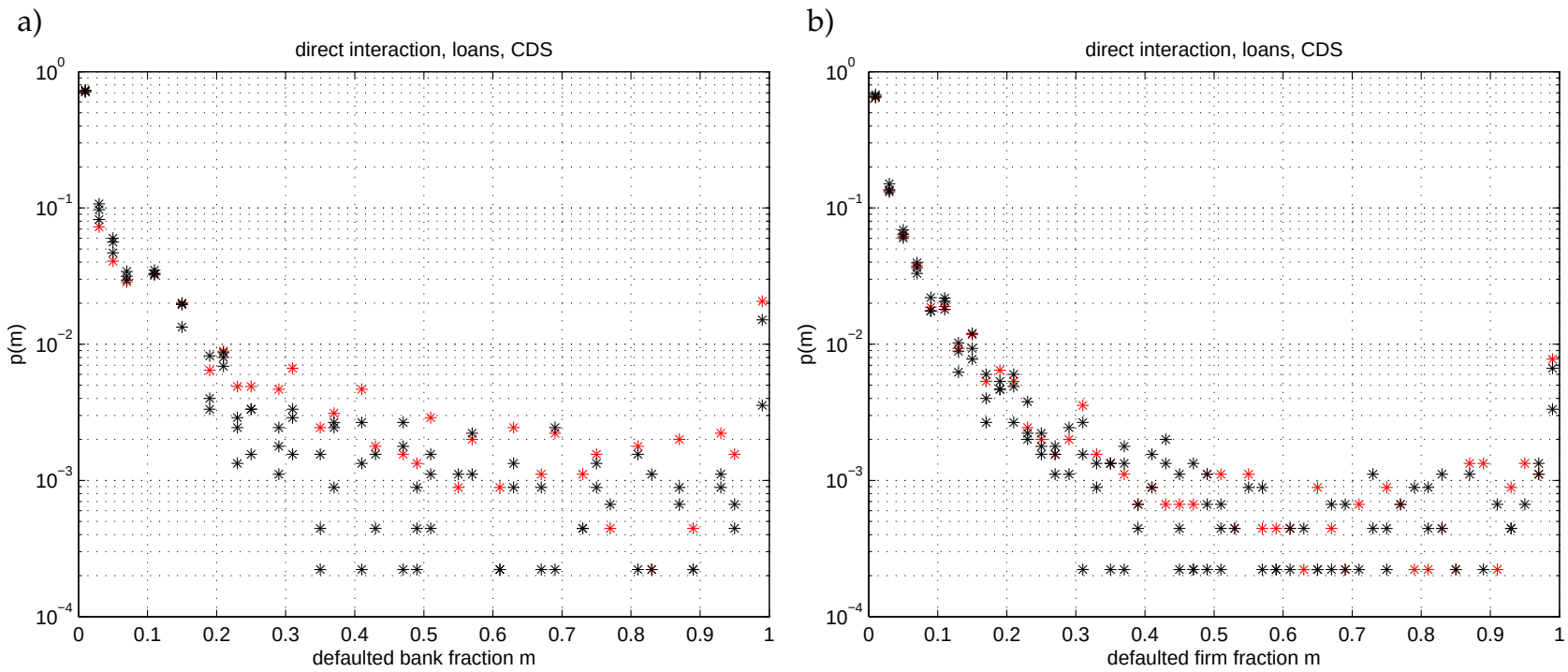
distribution of default fractions over 4500 runs with direct interactions :
a) in the banking sector
b) in the firms sector

What are we looking at ?



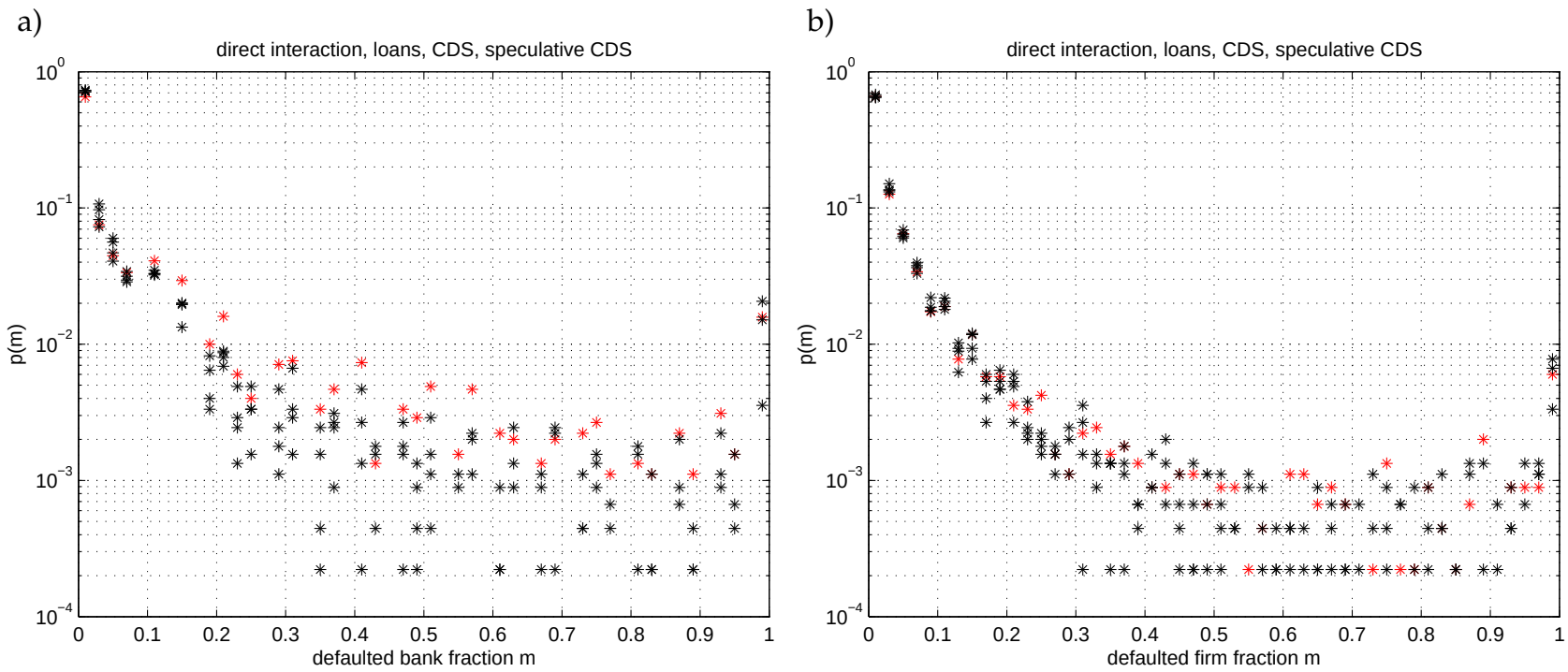
distribution of default fractions over 4500 runs with direct interactions and unhedged loans :
a) in the banking sector
b) in the firms sector

What are we looking at ?



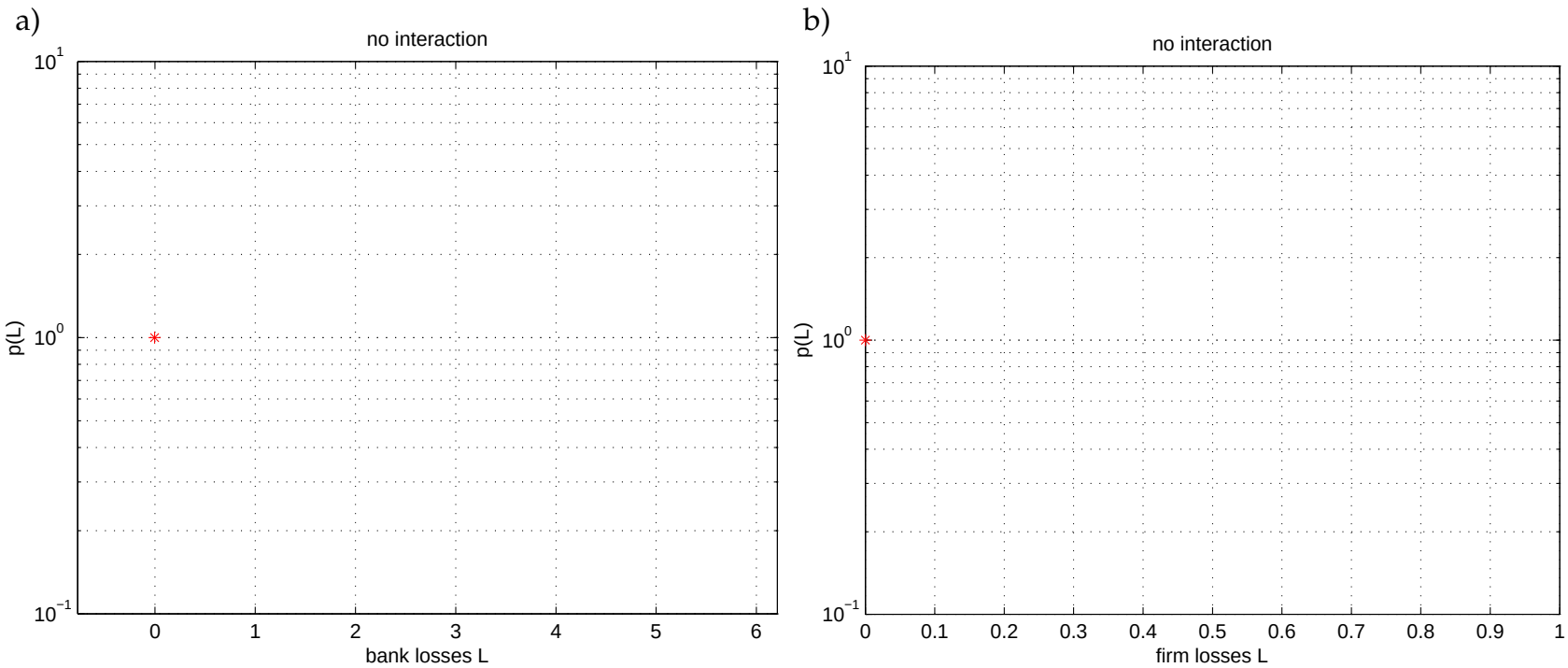
distribution of default fractions over 4500 runs with direct interactions, unhedged loans and CDS :
a) in the banking sector
b) in the firms sector

What are we looking at ?



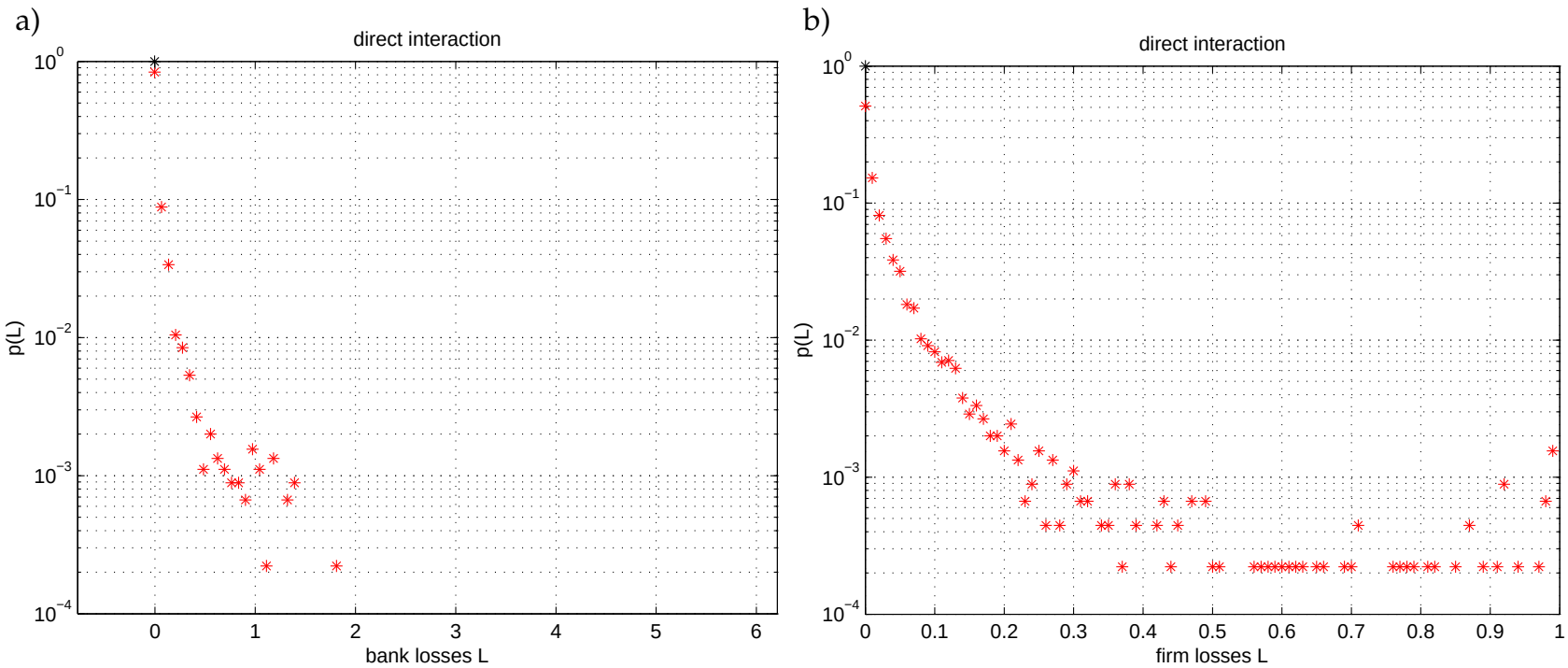
distribution of default fractions over 4500 runs with direct interactions, unhedged loans and speculative CDS :
a) in the banking sector
b) in the firms sector

What are we looking at ?



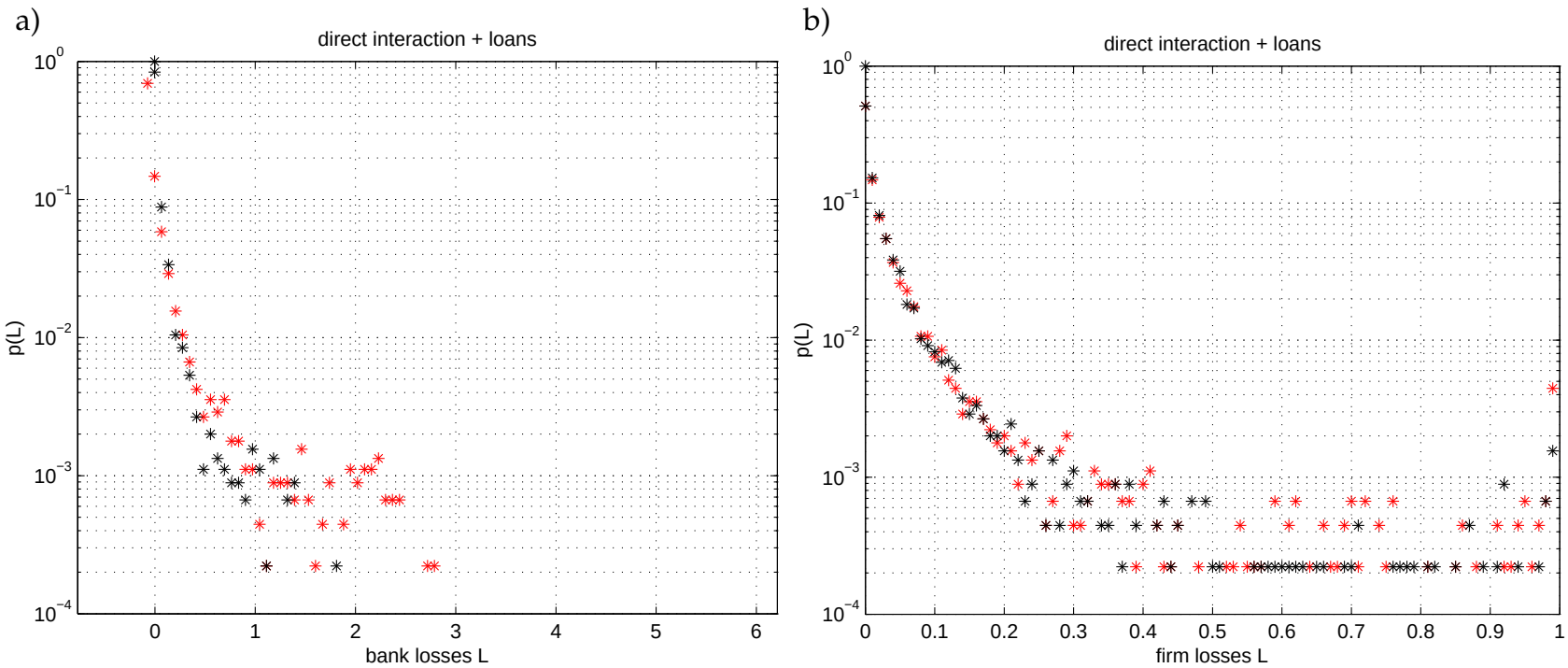
distribution of losses over 4500 runs in the absence of interactions :
a) in the banking sector
b) in the firms sector

What are we looking at ?



distribution of losses over 4500 runs with direct interactions :
a) in the banking sector
b) in the firms sector

What are we looking at ?



distribution of losses over 4500 runs with direct interactions and unhedged loans :
a) in the banking sector
b) in the firms sector

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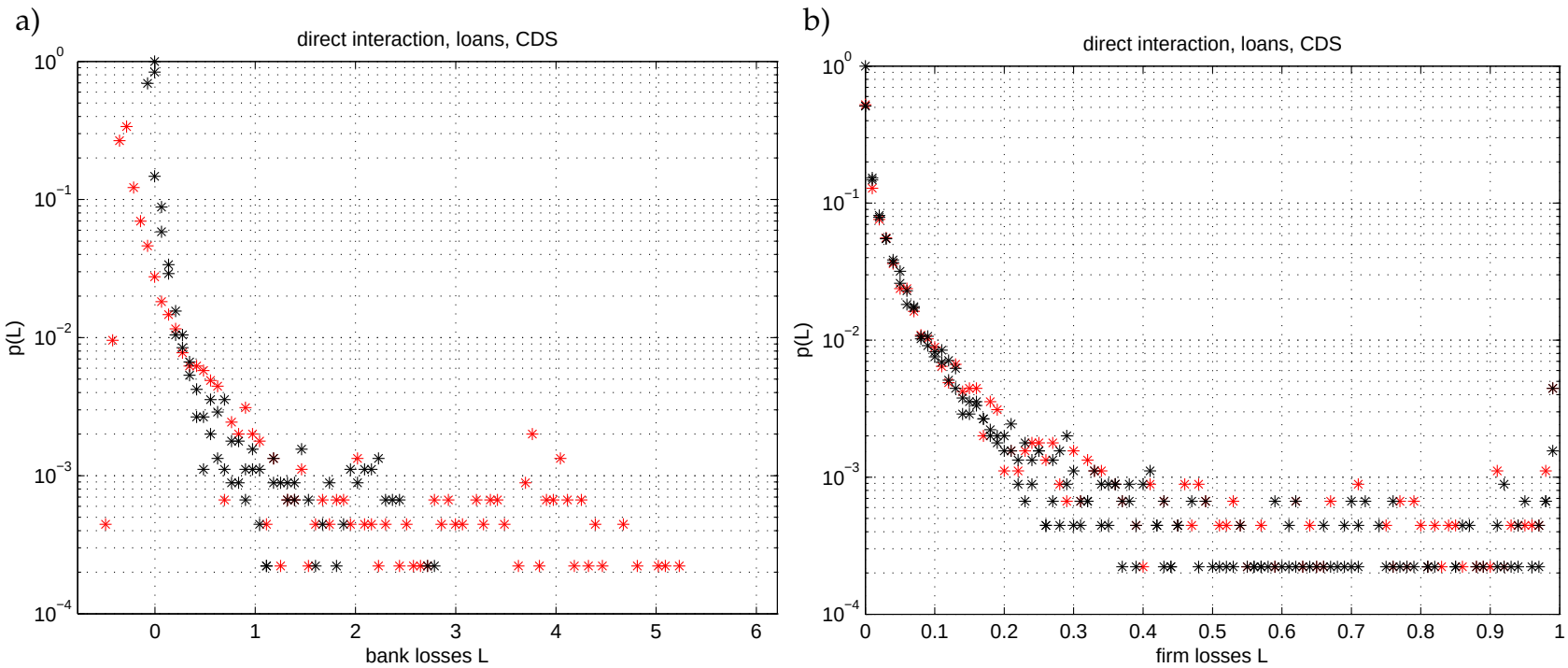

distribution of losses over 4500 runs with direct interactions, unhedged loans and CDS :
a) in the banking sector
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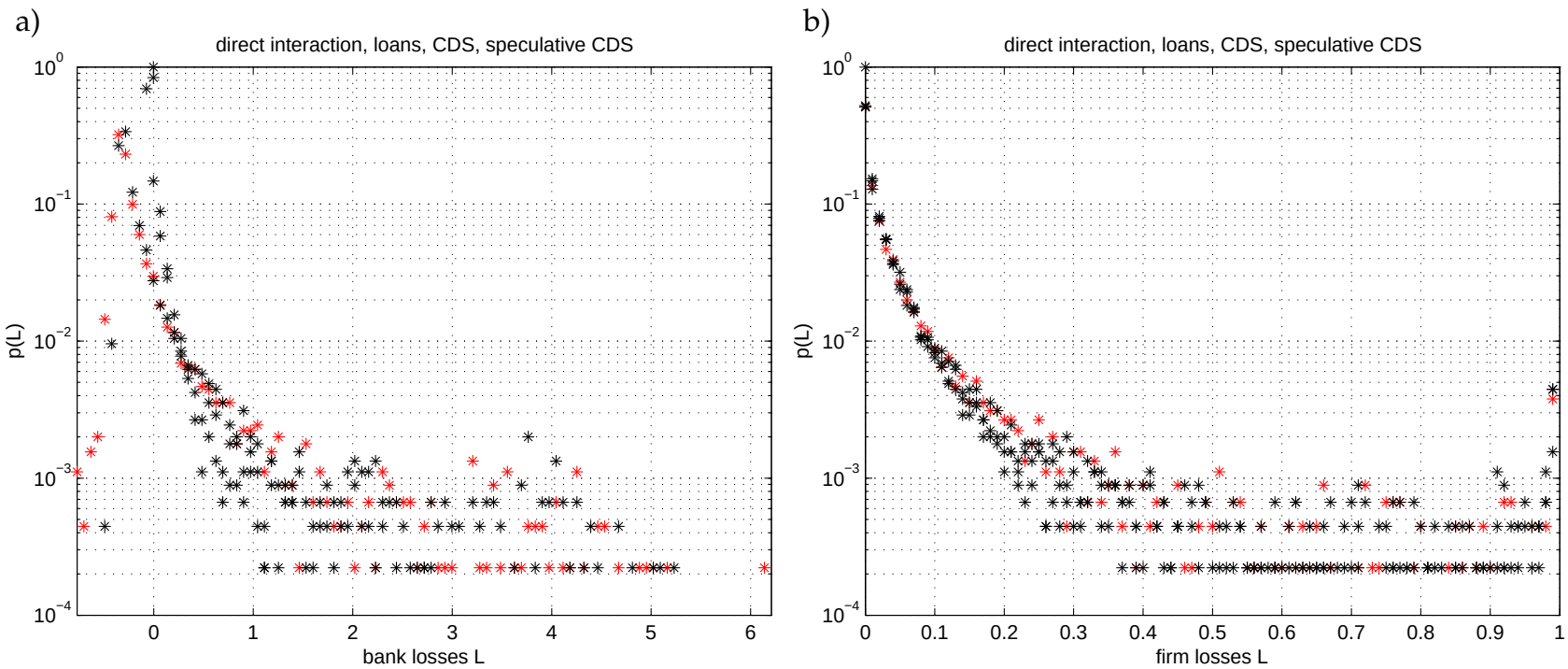

distribution of losses over 4500 runs with direct interactions, unhedged loans and speculative CDS :
a) in the banking sector
b) in the firms sector

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what comes next

- integrate equity requirements and liquidity shocks (fire sales...)
- integrate degree-degree correlations and wealth-degree correlations
- solve the model analytically

secondments

Secondments considered

1. ENS : Similar topics (contagion processes in networks)
2. ICTP : Similar topics (statistical physics approach to systemic risk)
3. NTNU : Complementary approach (inference of interactions from non-equilibrium data vs predictions from specified interactions)