NETADIS Kick off meeting

Torino ,3-6 february 2013

Payal Tyagi

PEOPLE

MARIE CURIE ACTIONS

Outline

- My background
- Project:

  1) Theoretical part

    a) Haus master equation and Hamiltonian

    b) First steps:

      - Modelling systems with disordered couplings
      - with multi body interactions (non- linear)
      - Cavity method

  2) Numerical part

    GPU based Monte Carlo numerical simulation

- Courses

My Background

--> Previous research experience:

Dr. Pranesh Sengupta, Materials Science Department,

Bhabha Atomic Research Centre (BARC), Mumbai :

"Crystal-chemical comparative study between naturally occurring sphene crystal".

EPMA, TEM, Raman microscopy, UV-VIS spectrography

--> MS Physics, Banasthali University (july 2010-may 2012) .

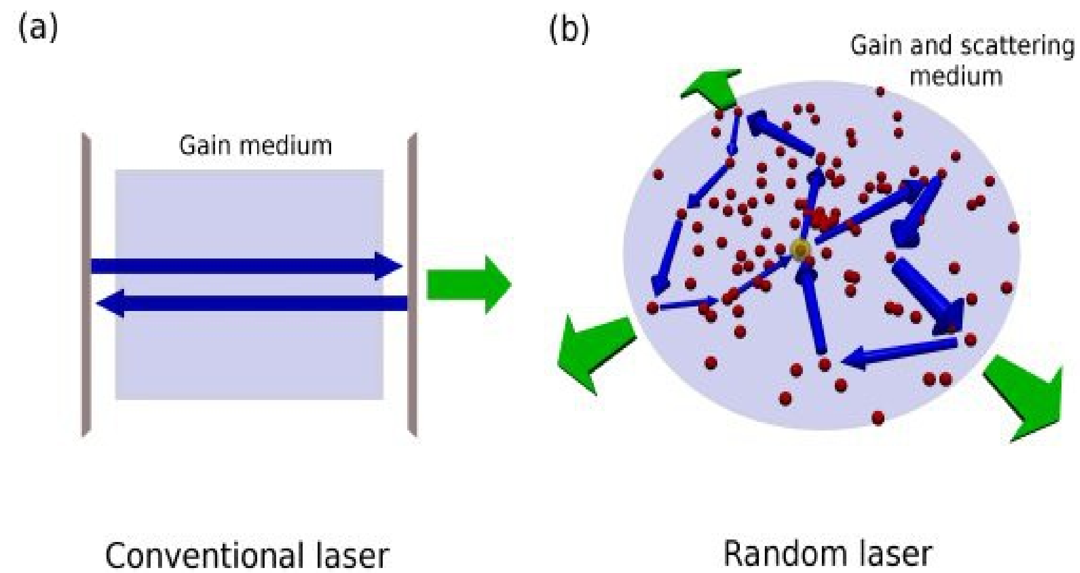
--> BS Physics, St. Stephen's College, University of Delhi (july 2007- may 2010) .

Project

Supervisor : Dr. Luca Leuzzi

Aim : "Inference of coupling of waves in non linear disordered medium."

- understanding random laser through disordered systems
- strong interaction in closed cavity
- random distribution of scatterers
- localization of em modes
- scattering -> degree of localization
- disorder induced random couplings
- non linearity couples the modes
- paramagnetic
- ferromagnetic
- complex glassy regimes
- statistical properties-> phase relations



C. Conti and L. Leuzzi "Complexity of waves in non linear disordered media", Phys Rev B 83, 134204(2011)
D.S. Wiersma , Nat Phys 4, 359 (2008)

Haus Master equation

Haus master equation for mode locking lasers in ordered cavities

$$T_R \frac{\partial a}{\partial t} = (g - l)a + \left(\frac{1}{\omega^2} + iD\right)\frac{\partial^2 a}{\partial \tau^2} + (\gamma - i\delta)|a|^2 a$$

Mode locking condition :

$$\omega_j + \omega_k - \omega_l - \omega_m = 0$$

Langevin dynamical equation :

$$\dot{a}(t) = \frac{-\partial H}{\partial a_n} + \eta_n(t)$$

$$\dot{a}(t) = (g_n - l_n + iD_n)a_n + (\gamma - i\delta)\Sigma_{\omega_j+\omega_k=\omega_l+\omega_m} a_j^* a_k a_l + \eta_n(t)$$

Hamiltonian :

$$H = -\Re\left[\sum_{j<k} G_{jk}^{(2)} a_j a_k + \sum_{\omega_j+\omega_k=\omega_l+\omega_m} G_{jklm}^{(4)} a_j^* a_k^* a_l a_m\right]$$

Where,

g_n : *gain coefficient*
l_n : *loss term*
D_n : *group velocity of the wave packet*
γ : *coefficient of saturable absorber*
δ : *coefficient of the Kerr lens*
$\eta_n(t)$: *white noise*

H. A. Haus, IEEE J. Quantum Electron . 6, 1173 (2000)

Hamiltonian

Amplitude : $\sum_i |a_i|^2 = \sum_i A_i^2 = E$, $a = A e^{i\phi}$

Hamiltonian for closed cavity:

$$H = \sum{}'^{1,N}_{\omega_j+\omega_k=\omega_l+\omega_m} G_{jklm} A_j A_k A_l A_m \cos(\phi_j + \phi_k - \phi_l - \phi_m)$$

$$H_J[\phi] = -\sum^{1,N}_{i_1<i_2, i_3<i_4} J_i \cos(\phi_{i_1} + \phi_{i_2} - \phi_{i_3} - \phi_{i_4})$$

Effective interaction among mode amplitudes is governed by:

$$G_{jklm} = 1/2 \sqrt{\omega_j \omega_k \omega_l \omega_m} \int_V d^3 r \chi^{(3)}_{\alpha\beta\gamma\delta}(\omega_j, \omega_k, \omega_l, \omega_m; r) E_j^\alpha(r) E_k^\beta(r) E_l^\gamma(r) E_m^\delta(r)$$

where ,

G_{jklm} : Coefficient of spatial overlap of electromagnetic fields of the modes

$\chi^{(3)}$: non linear susceptibility

C. Conti and L. Leuzzi "Complexity of waves in non linear disordered media", Phys Rev B 83, 134204(2011)

Theoretical part

Bethe Lattice we study two different mean field models of Bethe lattice and look into their mean field properties.

- **XY model** $$H_J[\phi] = -\sum_{i_1<i_2, i_3<i_4}^{1,N} J_i \cos(\phi_{i_1}+\phi_{i_2}-\phi_{i_3}-\phi_{i_4})$$ $$where,\ i=\{i_1,i_2,i_3,i_4\}$$
  - angle variable is the phase of a mode.
    - mode phases are represented by continuous spins.
      - 4 body interaction, couplings from short to long range

- **Spherical spin model**
  - fully connected $$H=-\sum_{i_1>..>i_p=1}^{N} J_{i_1\dots i_p}\sigma_{i_1\dots i_p}$$
    - Ising spins are replaced by real continuous variables σ_i , for p=4
    - spherical constraint to keep energy finite: $\Sigma_i \sigma_i^2 = N$, N-dim sphere of $\sqrt{N}$ radius .

- **Continuos variable-XORSAT**

These are the optimization problems where we study a diluted p=4 spin model (both spherical and XY).

M . Mezard, G. Parisi, M. A. Virasoro, " Spin glass Theory and Beyond", World Scientific Lecture Notes in Physics Vol. 9

First Steps

- **SK model**
- infinite range spin glass model
- fully connected
- Couplings J are random and non zero
- distribution of local fields is Gaussian

Replica Symmetric treatment:

$$Hamiltonian: \ H_J[s] = -\Sigma_{1 \le i \le k \le N} \ J_{i,k} s_i s_k \ - \ \Sigma_i \ h s_i$$

$$Free\ energy: \ f_n = -(\frac{1}{\beta} nN) \ln Z_n$$

$$Replicated\ partition\ function: \ (Z_J)^n = \Sigma_{s^1} \Sigma_{s^2} \ldots\ldots \Sigma_{s^n} \exp - \Sigma_{a=1}^n \beta H_J[s^a]$$

$$f = -\beta/4(1-q)^2 - \int_{-\infty}^{\infty} dz/(2\pi)^1/2 \exp(-z^2/2) \ln(2\cosh(\beta q^{1/2} z + \beta h))$$

$$Magnetisation: \ m = \int_{-\infty}^{\infty} dz/(2\pi)^{1/2} \exp(-z^2/2) \ \tanh(\beta q^{1/2} z + \beta h)$$

$$Order\ parameter: \ q = \int_{-\infty}^{\infty} dz/(2\pi)^{1/2} \exp(-z^2/2) \ \tanh^2(\beta q^{1/2} z + \beta h)$$

$$Entropy: \ S(0) = -0.17$$

- Replica Symmetry Breaking solution
- Order parameter : probability distribution function P(q):

$$P(q)=m\,\delta(q-q_0)+(1-m)\,\delta(q-q_1)$$

$$q^k=\lim_{n\to 0} 1/(n(n-1))/2\,\Sigma_{\{a,b\}}[Q_{a,b}]^k$$

Matrix :

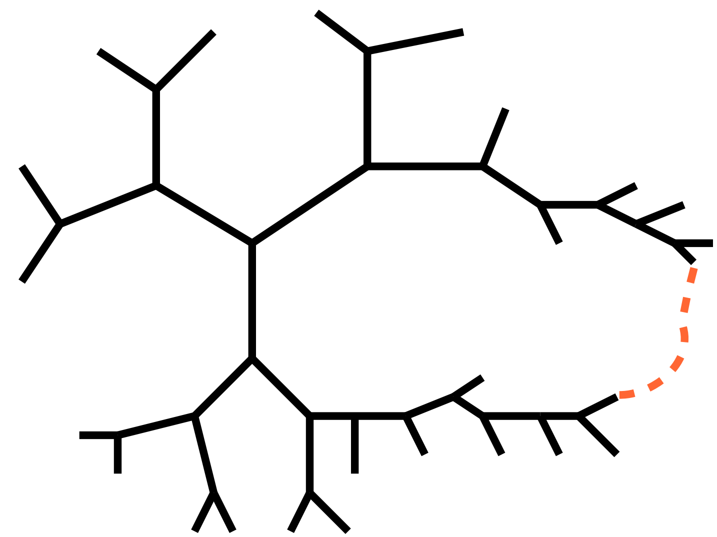
$$Q_{ab}=q_1 \quad if \quad I(a/m)=I(b/m)$$
$$Q_{ab}=q_0 \quad if \quad I(a/m)\neq I(b/m)$$

$$\textit{Free energy}:\ f(q_0,q_1,m)=-\beta/4[1+mq_0^2+(1-m)q_1^2-2q_1]-ln2$$
$$-\int dp_{q_0}(z)m^{-1}\ln\{\int dp_{(q_1-q_0)}(y)\cosh^m[\beta(z+y+h)]\}$$

$$q_1=\int dp_{q_0}(z-h)\int dp_{q_1-q_0}(y)\cosh^m[\beta(z+y)]\tanh^2(\beta(z+y))/D(z)$$

$$q_0=\int dp_{q_0}(z-h)\{\int dp_{q_1-q_0}(y)\cosh^m[\beta(z+y)]\tanh(\beta(z+y))/D(z)\}^2$$

Bethe Lattice



- Finite dimensional model
- System of N Ising spins, $\sigma_i = \pm 1, \quad i \in \{1, N\}$

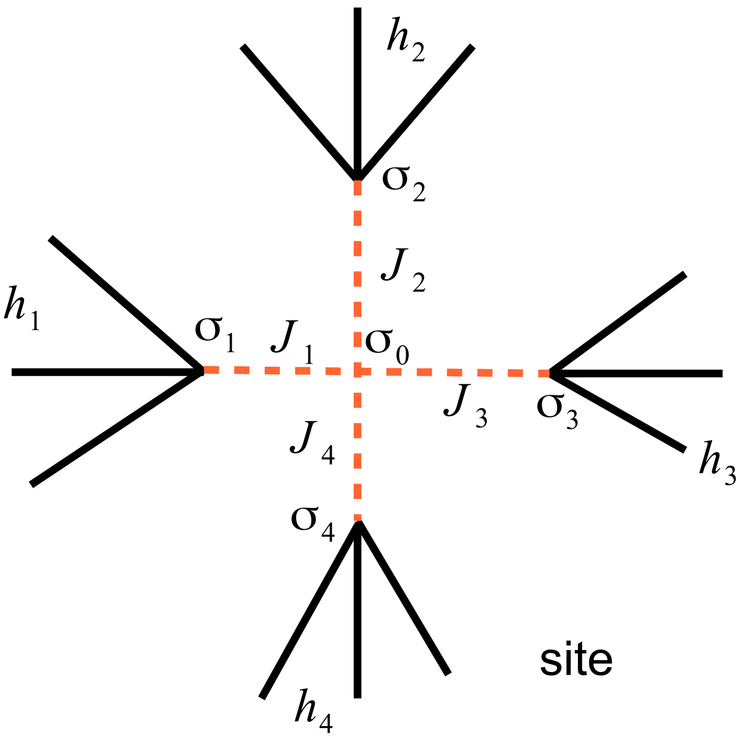
$$E = -\Sigma_{i,j} \, J_{i,j} \, \sigma_i \, \sigma_j$$

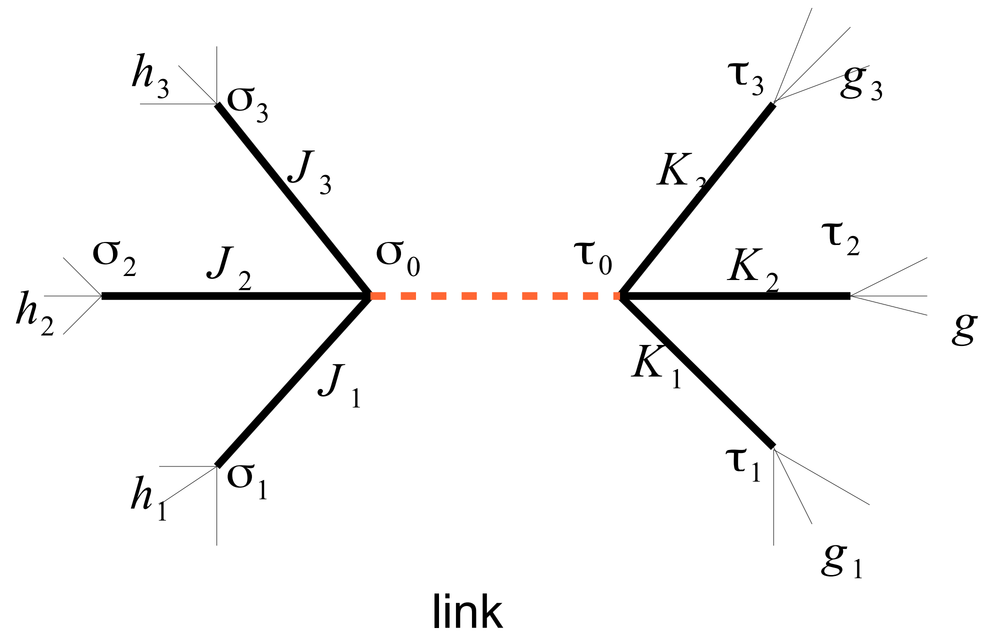
$J_{i,j}$: independent random variable with probability distribution P(J)

- Cayley tree:
  - inhomogeneous
  - 1st shell :at i=0, k+1 neighbours --> 2nd shell each spin connected to k new neighbours........ --> L'th shell (boundary).
- Random graph with fluctuating connectivity: Erdos Renyi graphs
  - link present : c/N and absent : 1- c/N
  - where c : number of links connected to a point with Poisson distribution
- Bethe lattice :
  -Random graph with fixed connectivity k+1
  - its a subset of Cayley tree containing only first few shells
  - statistically homogeneous
  - locally tree like, with fixed branching ratio
  - No small loops, size of loop ~ O (log N)

M. Mezard, G. Parisi, "The Bethe lattice spin glass revisited", Eur. Phys. J. B 20, 217 (2001)

Cavity Method





- mean field approach
- equivalent to Replica approach
- iterative method on tree like structures
- comparison of a N --> N+1 spin system.
- adding a new spin at site 0.
- site and link contribution.
- reshuffling of spins.
- reweighing of configurational weights for contribution to new equilibrium state

Cavity method equivalent to 1RSB

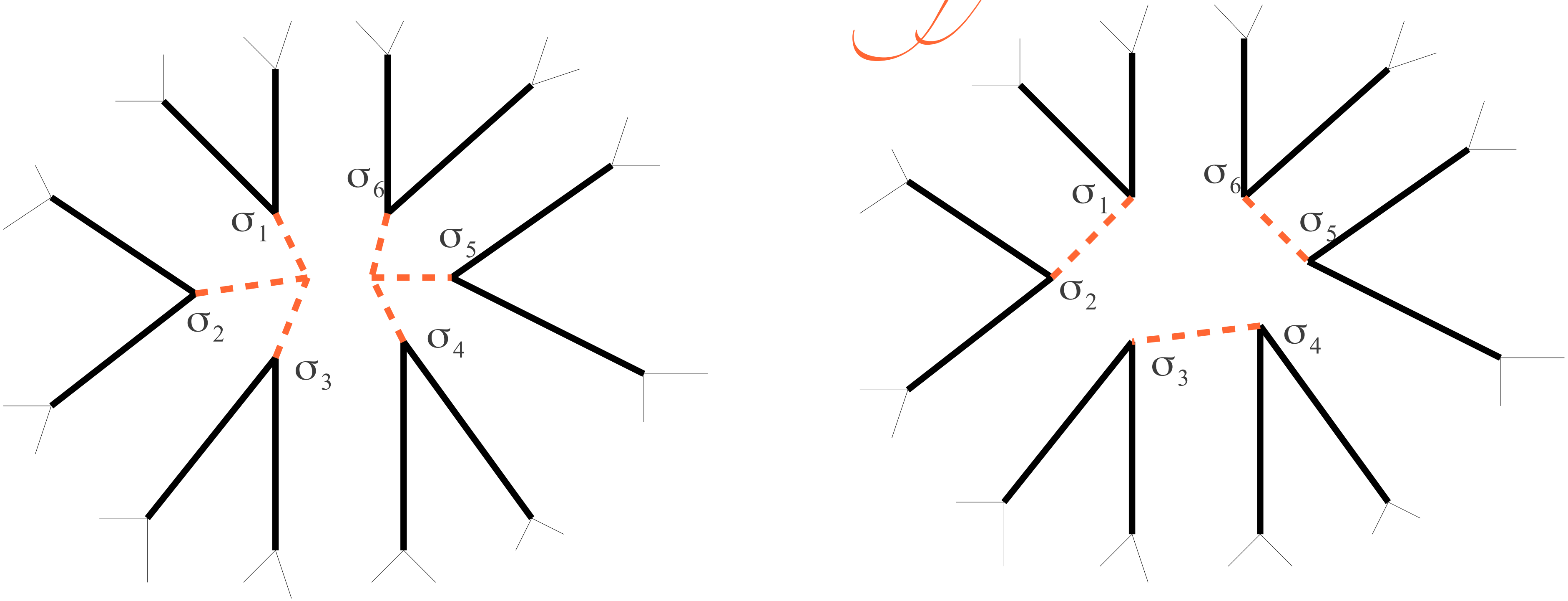
- Existence of several pure α : $1 \rightarrow \infty$
- merge k sites to a site i=0 ,

  field at i=0 : h_0^{α} , free energy iid exponential distribution : F^{α}

  Initially various states have weights : $W^{\alpha}=\frac{\exp(-\beta F^{\alpha})}{\sum_{\gamma}\exp(-\beta F^{\gamma})}$
- Field at i=0 calculated as: $h_0=\sum_{i=1}^{k} u(J_i, h_i)$ $u(j,h)=1/\beta\, atanh(\tanh(\beta J)\tanh(\beta h))$
- states are ordered in increasing order of free energy and only lowest M states are considered.
- When we change the site i, the probability distribution of fields fluctuates and the new probability distribution Q(h) is to be calculated.
- N local fields h_i of which k fields are chosen, $i_1,..,i_k$ *of* $1,..,N$

  h_i^{α}, *where* $i \in 1,....,N$, $\alpha \in 1,....,M$

Free Energy



$$F = \frac{k+1}{2} \Delta F^{(link)} - \Delta F^{(site)}$$

Site contribution: $-\beta \Delta F^{(site)}(J_1 .. J_{k+1}, h_1 .. h_{k+1}) = \ln[2\cosh(\beta \Sigma_{i=1}^{k+1} u(J_i, h_i))] + \Sigma_{i=1}^{k+1} \ln[\frac{\cosh \beta J_i}{\cosh(\beta u(J_i, h_i))}]$

Link contibution: $-\beta \Delta F^{(link)}(J_1 .. J_k, K_1 .. K_k, h_1 .. h_k, g_1 .. g_k) = \Sigma_{i=1}^{k} \ln \frac{\cosh(\beta J_i)\cosh(\beta K_i)}{\cosh(\beta u(J_i, h_i))\cosh(\beta u(K_i, g_i))}$

$+\ln(\Sigma_{\sigma_0, \tau_0}(\beta J_0 \sigma_0 \tau_0 + \beta \sigma_0 \Sigma_{i=1}^{k} u(J_i, h_i) + \beta \tau_0 \Sigma_{i=1}^{k} u(J_i, h_i) + \beta \tau_0 \Sigma_{i=1}^{k} u(K_i, g_i)))$

Site contribution :

$$Field:\ h_0 = \Sigma_{j=1}^{k+1} u(J_j, h_j)$$

$$Free\ energy:\ F^{(1)} = \frac{-1}{\beta} x \ln\left(1/M\, \Sigma_\alpha \exp[-\beta x \Delta F^\alpha]\right)$$

$$Order\ parameter:\ q_1 = \frac{1}{M} \Sigma_\alpha \tanh^2(\beta h^\alpha)$$

$$Order\ parameter:\ q_0 = \frac{1}{M(M-1)} \Sigma_{\alpha\neq\gamma} \tanh(\beta h^\alpha) \tanh(\beta h^\gamma)$$

$$x\ derivative:\ d^{(1)} = \frac{1}{x} \frac{\Sigma_\alpha \exp(-\beta x \Delta F^\alpha) \Delta F^\alpha}{\Sigma_\alpha \exp(-\beta x \Delta F^\alpha)}$$

Link contribution:

$$Fields:\ h_0^\alpha = \Sigma_{i=1}^{k} u(J_i, h_i)\ ,\qquad g_0^\alpha = \Sigma_{i=1}^{k} u(k_i, g_i)$$

$$Free\ enrgey:\ F^{(2)} = \frac{-1}{\beta} x \ln\left(1/M\, \Sigma_\alpha \exp[-\beta x \Delta F^\alpha]\right)$$

$$Order\ parameter:\ q_0^{(l)} = \frac{1}{M} \Sigma_\alpha \tanh^2(\beta v^\alpha)$$

$$Order\ parameter:\ q_1^{(l)} = \frac{1}{M(M-1)} \Sigma_{\alpha\neq\gamma} \tanh(\beta v^\alpha) \tanh(\beta v^\gamma)$$

$$x\ derivative:\ d^{(2)} = \frac{1}{x} \frac{\Sigma_\alpha \exp(-\beta x \Delta F^\alpha) \Delta F^\alpha}{\Sigma_\alpha \exp(-\beta x \Delta F^\alpha)}$$

Numerical part

- finite dimensional model , finite number of modes in a given sample induced geometry.

  => simulations.

- Intensity spectra of random laser--> space and frequency correlations among amplitude and phases.
- Continuous variables --> Monte Carlo simulations --> GPU's
- Bethe lattice was mean field model --> modes distance independent.
- When distance taken into consideration --> computations are complex.
- Levy graph :

$$P_{ij} = \frac{1}{|\vec{i} - \vec{j}|^{\rho}}$$

$$i - j : \ \textit{distance between the sites}$$

$$\rho \gg 1 : \textit{short range}$$

$$\rho = 0 : \textit{Bethe lattice}$$

Further developments

- As a further step--> apart from random lasers --> study interaction between radiative modes in photonics (linear interaction and disorder).
- Inverse problem --> four point correlation --> recover interaction between modes from experimental data --> correlations between modes.

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Also, possiblity of secondment in generalisation of inverse problem methods to continuous variables and 4- point correlation

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Torino

Courses

1) Coursework in Sapienza University (January- July) :

- **Structural glasses**
- **Spin glasses**
- **Random graphs**
- **Monte Carlo method**

2) Course on **GPGPU** and **CUDA** programming, CINECA Bologna (9-10 May).

3) 22nd summer school on **Parallel computing**, CINECA Bologna (20-31 May).

4) International summer school, fundamental problems in **Statistical Mechanics** XIII, Leuven, Belgium (16-29 June).

5) Advanced Workshop on **Nonlinear Photonics, Disorder and Wave Turbulence** ICTP, Trieste (15-19 July).

Thank you