The Abdus Salam
International Centre for Theoretical Physics

Modeling financial markets

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Outline

- What are markets good for?
- Information efficiency
  - Statistical mechanics of financial markets: the Minority Game
  - Market impact of meta-orders
- Risk and correlations
- Inverse statistical mechanics: fitting models to data

What are markets good for?

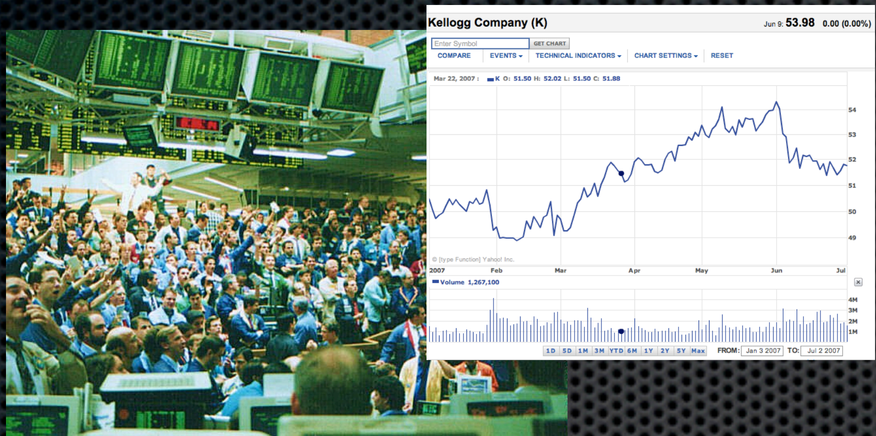
(individual optimum) x N $\neq$ *global optimum*

- markets allocate optimally resources
  It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest.
  (A. Smith)
- markets incorporate efficiently available information in prices
- markets allow individuals to cope with uncertainty and reduce risk

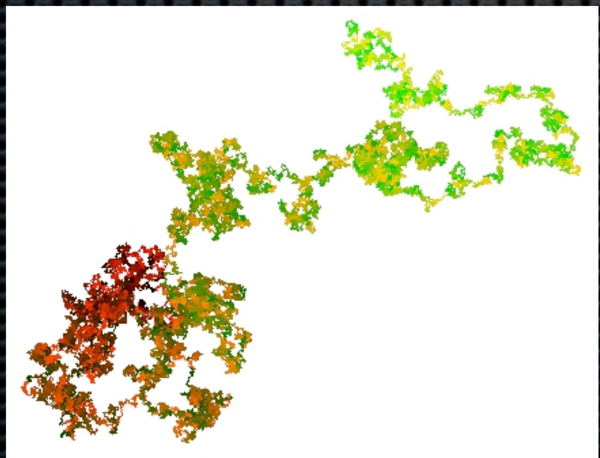
Information efficiency

Markets are very complex but price behavior is very simple

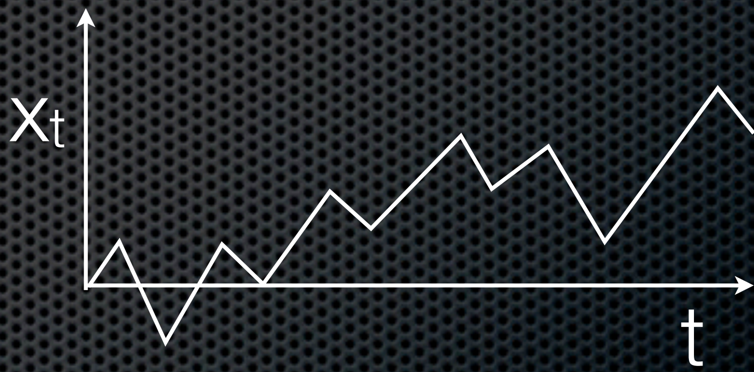
- The dynamics of prices (Bachelier 1900)



- Brownian motion (Einstein 1905)



The random walk

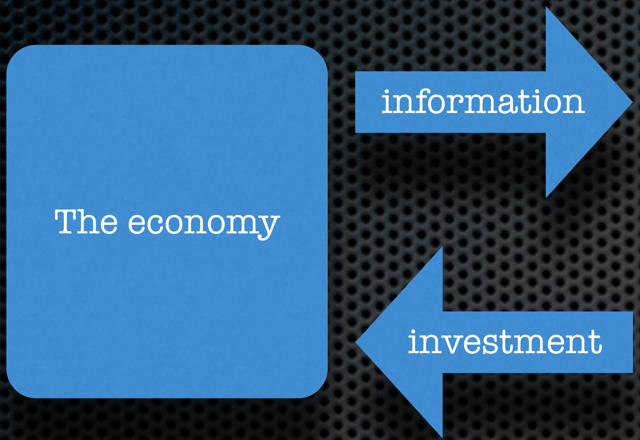


toss a coin at each step

$$x_{t+1} = \begin{cases} x_t + 1 & \text{if head} \\ x_t - 1 & \text{if tail} \end{cases}$$

Market information efficiency

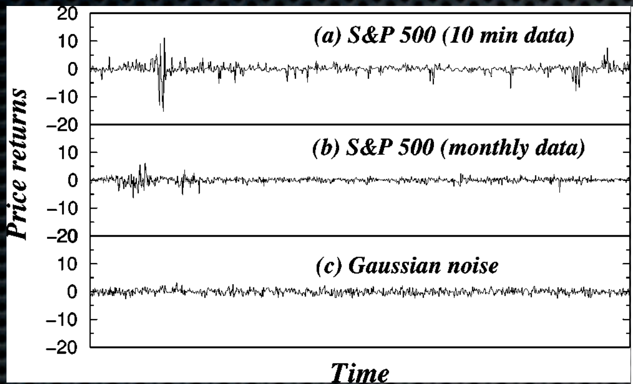
- Random walks as a result of strong interaction:

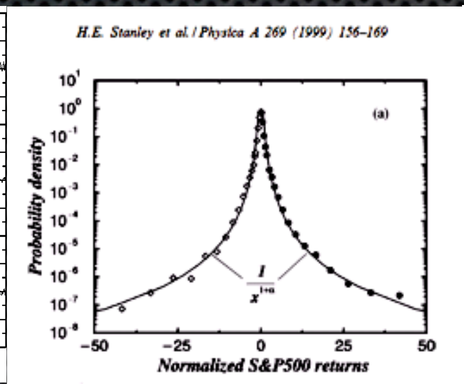


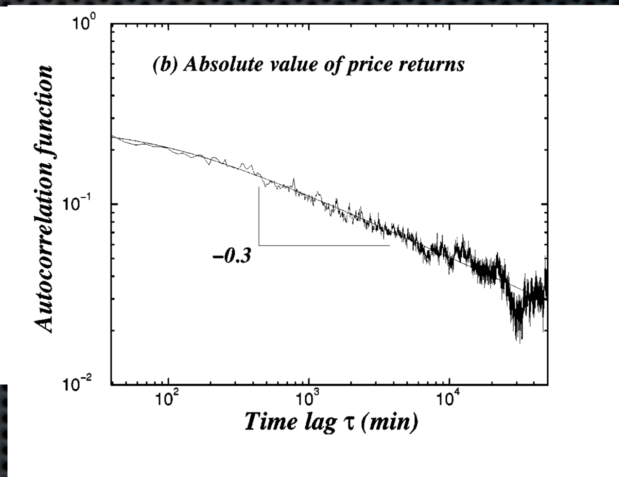
- A market is information efficient wrt information set, if prices would not change when that information set is revealed
- Prices are unpredictable because all causally meaningful information is exploited

But...

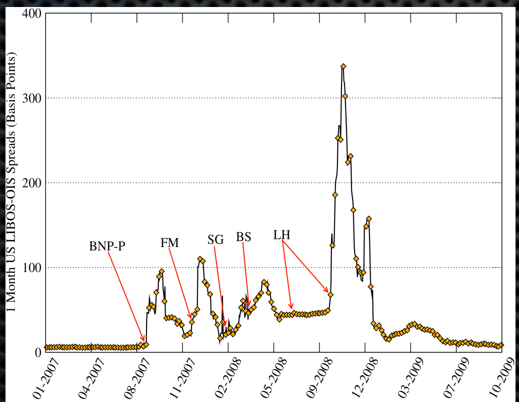
- Prices are not random walks (Mandelbrot 1965, Mantegna & Stanley 1995, etc)

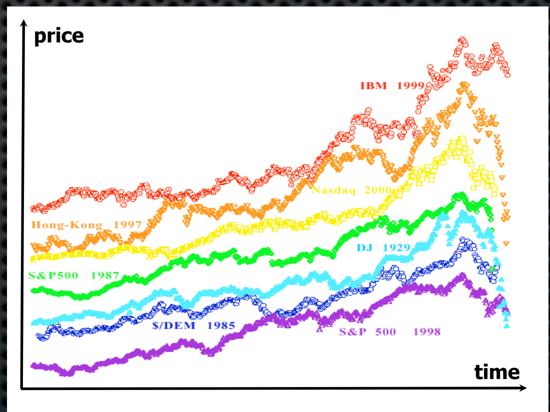






- Crashes and bubbles do occur





Bubbles in high-tech, real estate, commodities, oil, credit derivatives, food markets...
(Sornette, Woodard 2010)

In order to understand why markets fail, we need models that explain why they work

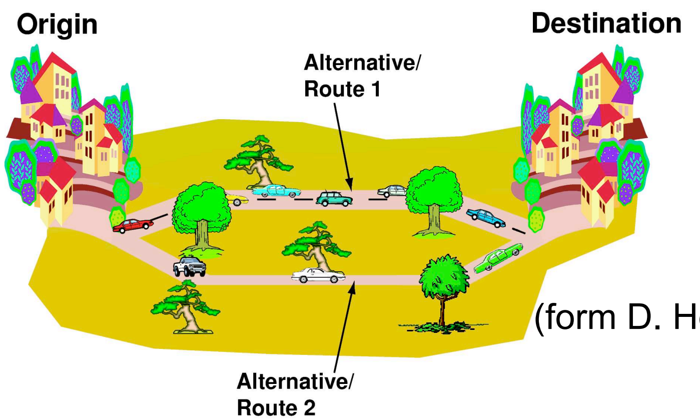
- Need to understand:
  - How do traders, seeking profit, make markets informationally efficient
  - How does traders' interaction in the "space of trading strategies" shape market ecology (information food web)
- How to deal with complexity?

A simple model of financial speculation

- a=+1 buy 1$
- a=-1 sell 1/p(t) shares
- demand = supply p(t+1)

$$\frac{p(t+1)-p(t)}{p(t)} = \frac{2A(t)}{N-A(t)} \qquad A(t)=\sum_{i=1}^{N} a_i(t)$$

- Optimal to buy (a=+1) when most sell (A<0) and viceversa
- Also N drivers 2 routes



(form D. Helbing 2003)

Game theory: Optimal behavior

Choice $a_i = \pm 1, \quad i = 1, \ldots, N$

$$u_i(a_i, a_{-i}) = \frac{N - a_i A}{2}, \qquad A = \sum_{j=1}^{N} a_j$$

Nash equilibria:

$$\begin{cases} a_i = +1 & \times \quad k \\ a_i = -1 & \times \quad k \\ \pi_{i,a} = \frac{1}{2} & \times \quad N - 2k \end{cases}$$

$$\#\mathrm{Nash} = 2 \sum_{k \leq N/2} \binom{N}{k} \approx 2^N$$

Q: will agents learn to converge to a Nash equilibrium?

Q: if yes, which one?

Q: what type of information should one give to agents to achieve optimal resource use?

$$\mathrm{efficiency} \approx \langle A^2 \rangle = N - 2k \qquad \mathrm{predictability} \approx \langle A \rangle^2 = 0$$

Learning dynamics

- Scores:

$$y_{i,a}(t+1) = y_{i,a}(t) + \frac{\Gamma}{N}\frac{N - aA(t)}{2}$$

$$A(t) = \sum_{j=1}^{N} a_j(t)$$

- Choice:

$$\pi_{i,a}(t) = \frac{e^{y_{i,a}(t)}}{e^{y_{i,a}(t)} + e^{y_{i,-a}(t)}}$$

- Simplification:

$$z_i(t) = y_{i,a=+1}(t) - y_{i,a=-1}(t)$$

$$z_i(t+1) = z_i(t) - \frac{\Gamma}{N}A(t)$$

$$\frac{1}{N}A(t) \simeq \langle a_i(t) \rangle = \tanh[z_i(t)/2], \qquad z_i(t=0) = 0$$

Excess volatility

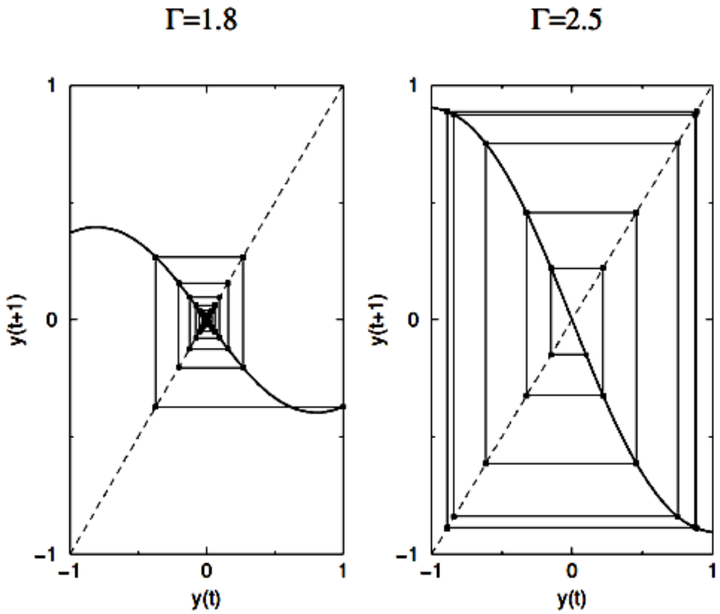
- Agents do not learn to play a Nash equilibrium
- Stationary state depends on
  - learning rate

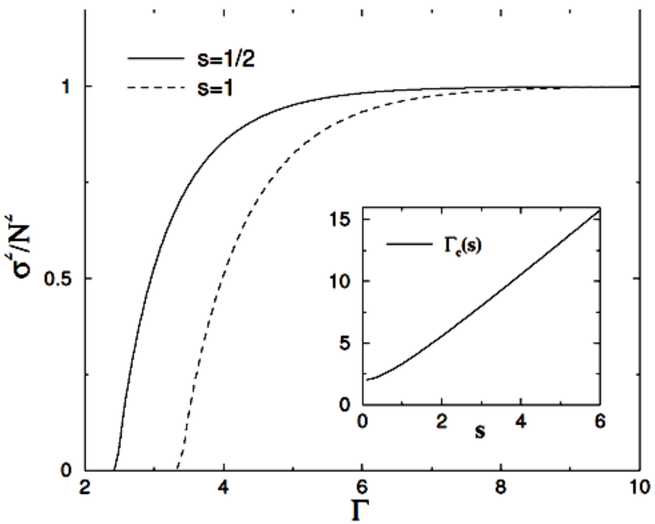
$$\Gamma < \Gamma_c \quad \Rightarrow \quad \sigma^2 = \langle A^2 \rangle \sim N$$
$$\Gamma > \Gamma_c \quad \Rightarrow \quad \sigma^2 = \langle A^2 \rangle \sim N^2$$

  - initial conditions (non ergodic behavior)

$$\sigma^2 \searrow \text{spread of initial conditions}$$

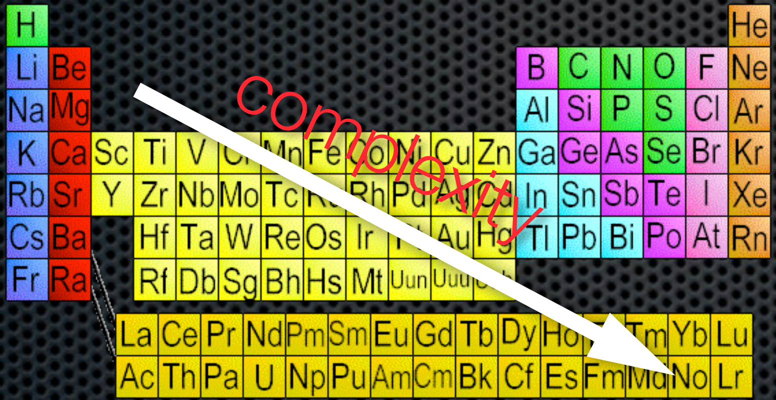
(note: initial conditions = prior beliefs)



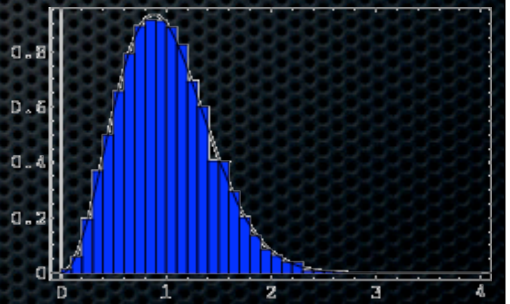


Traders in financial markets are heterogeneous!

- Wigner and heavy ions (1955)



$\hat{H} \approx$ random matrix

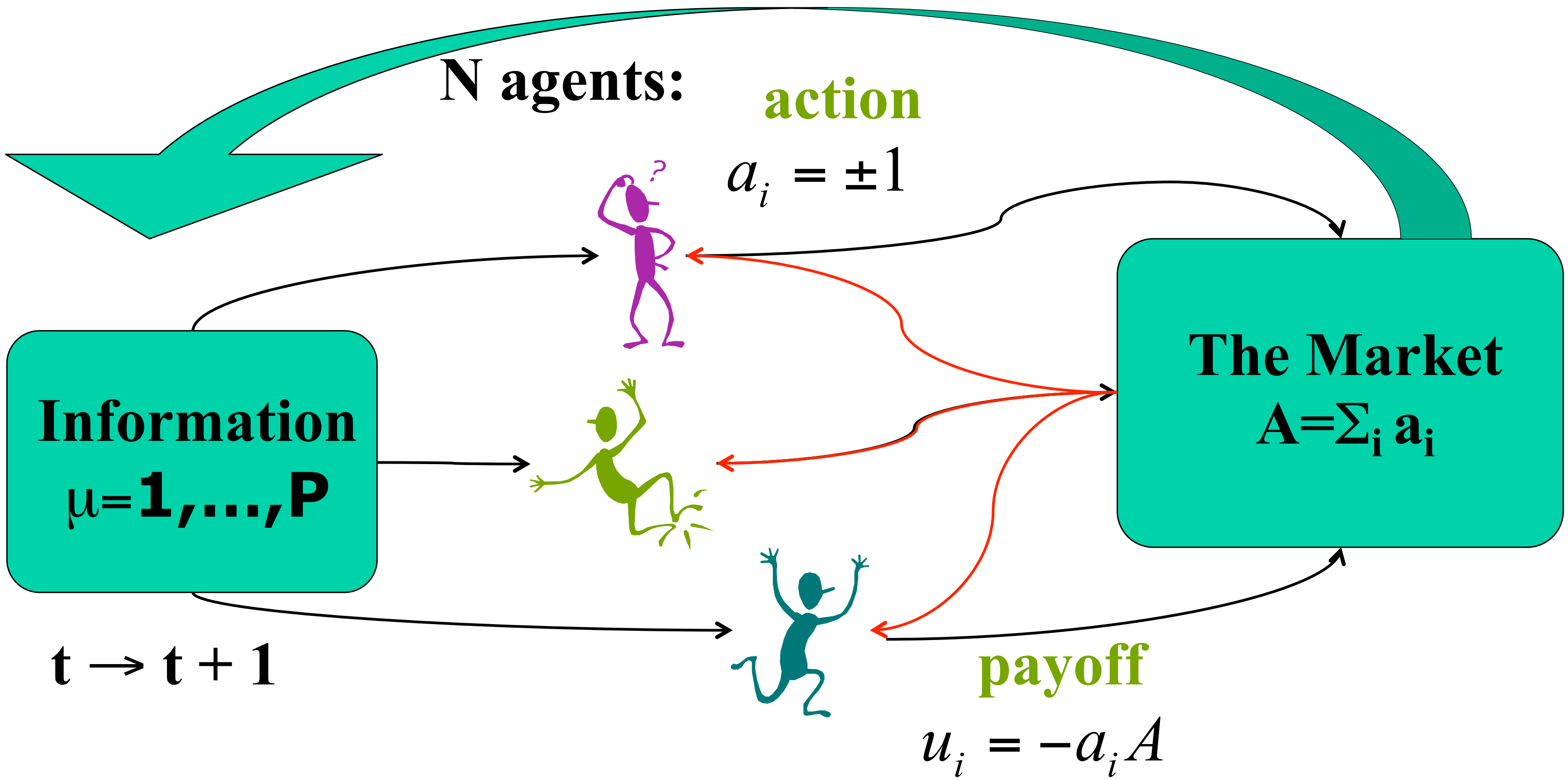


- Statistical mechanics of large random systems display self-averaging behavior (aka law of large numbers)

- Everything should be made as simple as possible, but not simpler (A. Einstein)

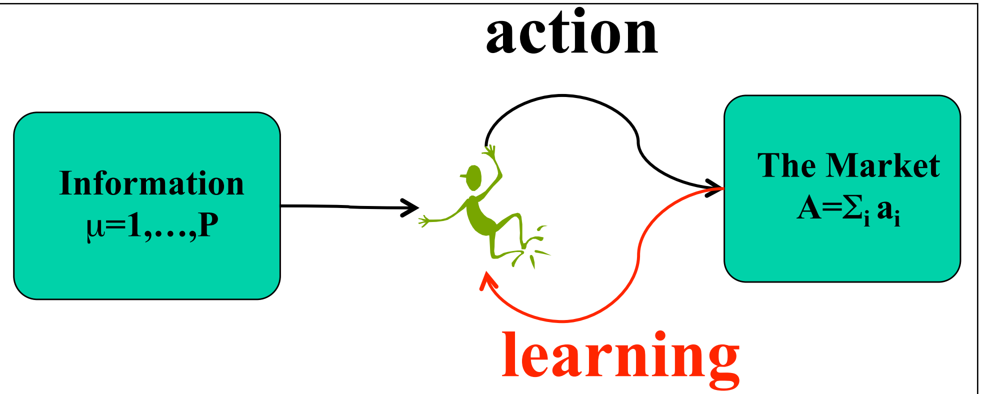
The minority game:

(Challet & Zhang 1997)



- Minority rule
- Agents = set of trading strategies $\{a_i: \mu \rightarrow a_i^\mu\}$ +
  learning and adaptation
- All agents are different (different strategies)
- Information is created by agents themselves (feedback)

2) adaptive agents:



- <u>Trading strategies:</u> $a^{\mu}_{s,i}, \quad s = 0,1,\ldots,S_i$
  - S active strategy (s ≠ 0): $a^{\mu}_{s\neq 0,i} = \pm 1$ random $\forall i,\mu$ and $s \neq 0$
  - 1 inactive strategy (s = 0): $a^{\mu}_{0,i} = 0 \ \forall i,\mu$
- <u>Learning:</u>

reward for not trading

$$U_{s,i}(t+1) = U_{s,i}(t) - a^{\mu}_{s,i}A(t) + \varepsilon_i \delta_{s,0}, \quad A(t) = \sum_{j=1}^{N} a^{\mu}_{s_j(t),j}$$

- <u>Choice:</u>

$$P\{s_i(t) = s\} \propto e^{\Gamma U_{s,i}(t)}$$

Very easy to run simulations:

```
...
μ=P*ran()+1
do I=1,Ns
        choice(I)=0
        do σ=1,S
                if (U(I,σ).lt.U(I,choice(I))) choice(I)=σ
        end do
end do
A=Ap(μ)    ! contribution from deterministic traders
do I=1,Ns
        A=A+a(I,μ,choice(I))
end do
do I=1,N
        U(I,0)=U(I,0)+ε
        do σ=1,S
                U(I,σ)=U(I,σ)-a(I,μ, σ)*A
        end do
end do
...
```

Numerical results:

- scaling α=P/N

(Savit et al PRL 1998)

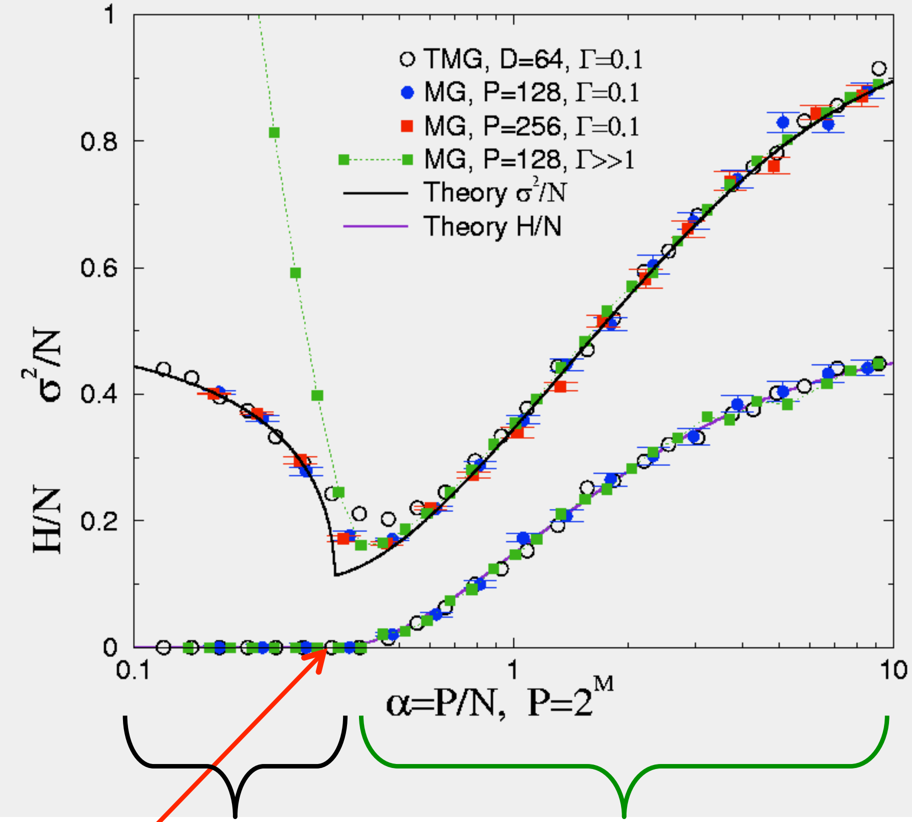
- Global efficiency

$$\sigma^2 = \left\langle A^2 \right\rangle = -\sum_{i=1}^{N} \left\langle u_i \right\rangle$$

- **Predictability**

$$\left\langle A \middle| \mu \right\rangle \neq 0 \quad \Rightarrow$$ **predictable**

$$H = \frac{1}{P} \sum_{\mu=1}^{P} \left\langle A \middle| \mu \right\rangle^2$$



Information efficient phase H=0

Information inefficient phase H>0

Phase transition

(Challet & Marsili 1999)

The stationary state is the solution of

$$\min_{\{m_i\}} H\{m_i\}$$

$$\left(m_i = \langle s_i \rangle\right)$$

indeed $$\frac{d\langle y_i \rangle}{d\tau} = -\overline{\xi_i \Omega} - \sum_{k=1}^{N} \overline{\xi_i \xi_k} m_k \quad \Rightarrow \quad \frac{dH}{d\tau} \leq 0 \quad \left(\langle y_i \rangle \nearrow m_i\right)$$

replica method

$$Z = \int d\{m\}\, e^{-\beta H\{m\}}$$

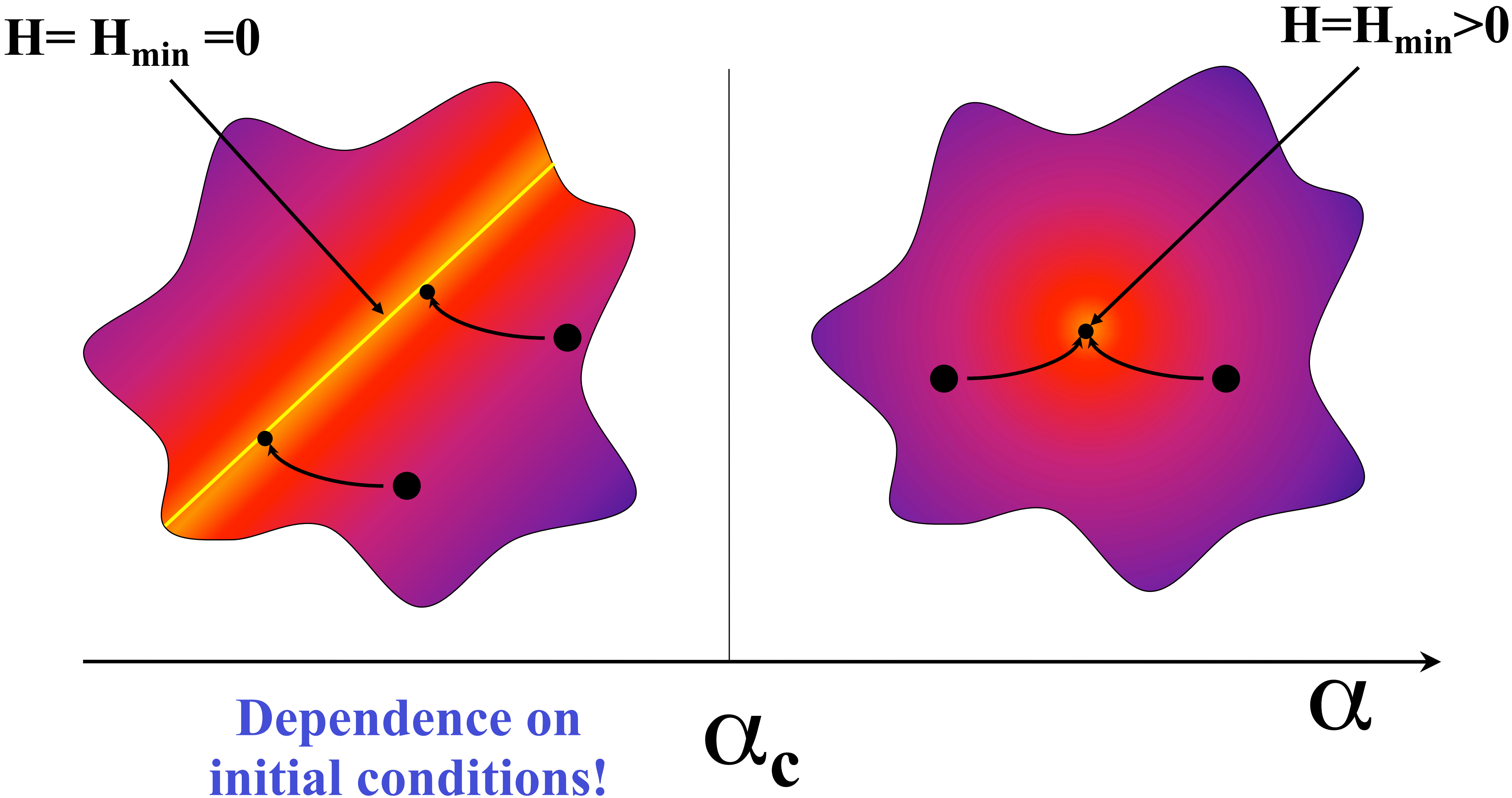
$$\min_{\{m\}} H\{m\} = \lim_{\beta\to\infty} -\frac{1}{\beta}\langle \log Z \rangle$$

$$\langle \log Z \rangle = \lim_{n\to 0} \frac{\langle Z^n \rangle - 1}{n}, \qquad Z^n = \int d\{m_1\}\cdots d\{m_n\}\, e^{-\beta[H\{m_1\}+\ldots+H\{m_n\}]}$$

… full pdf

Phase transition

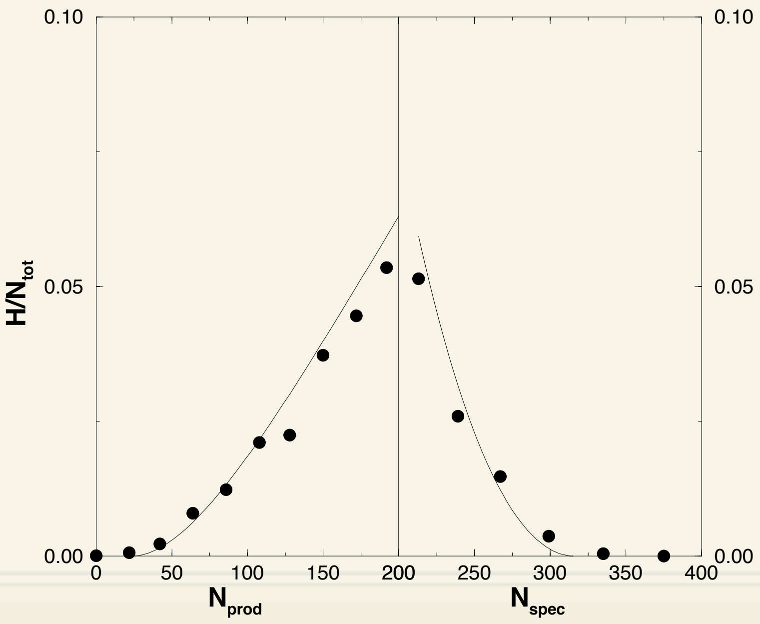
Density plot of H in the space $\{m_i\}$

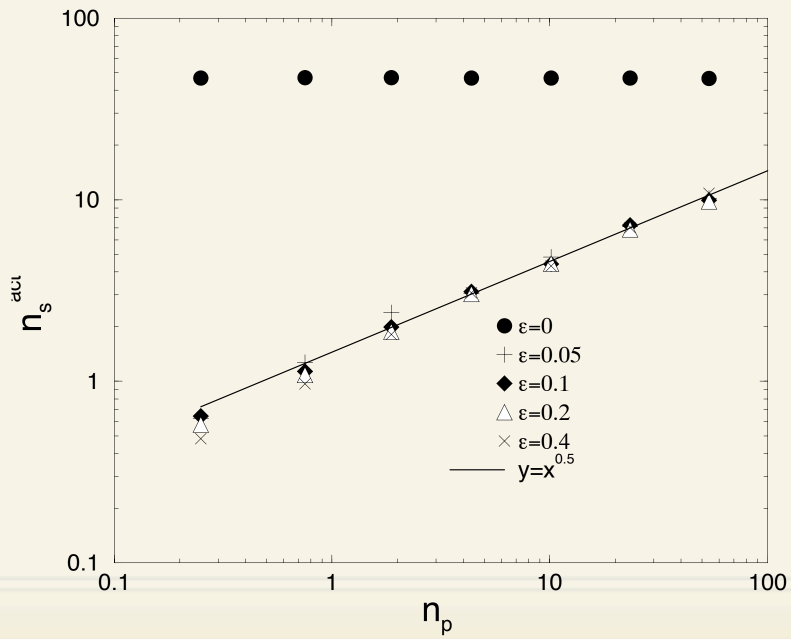


Why should agents trade?

- Market information ecology (Challet et al. 2001)
  - N_s Liquidity providers: play only if E[gain] > ε
  - N_p Liquidity takers (1 strategy)

$$N_s^{\text{active}} \sim \sqrt{N_p}$$

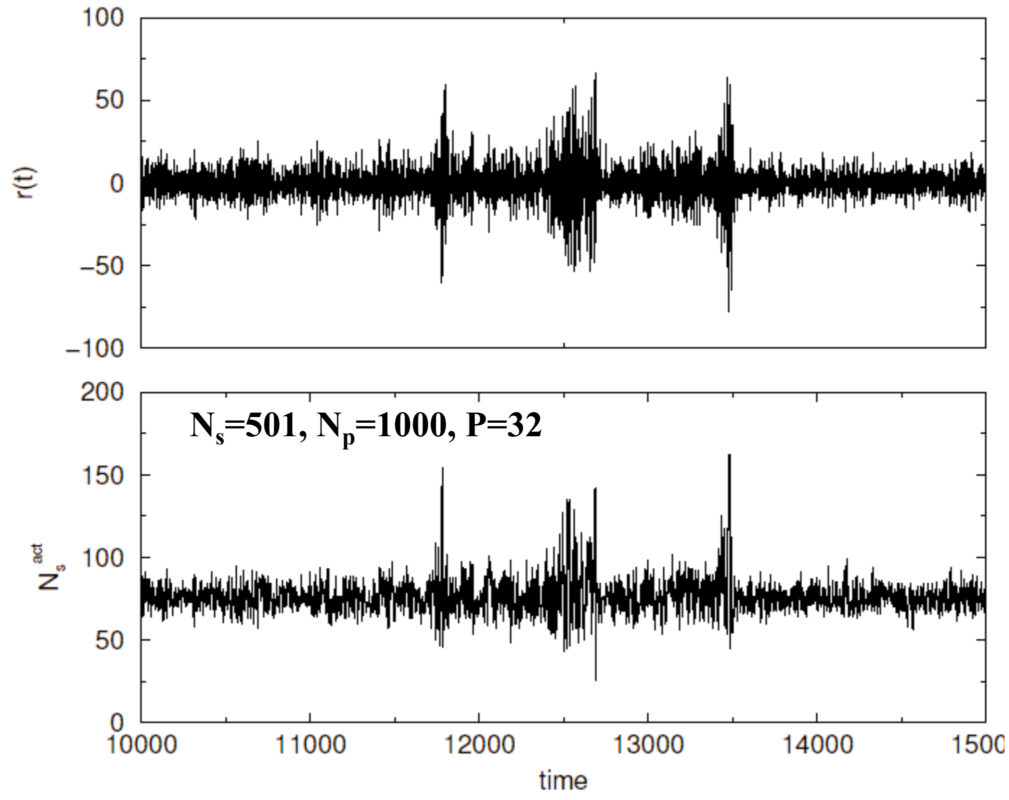




Close to where H~0:

(Challet, Marsili Zhang Physica A 2001)

Price dynamics: $\log p(t+1) = \log p(t) + \lambda A(t)$



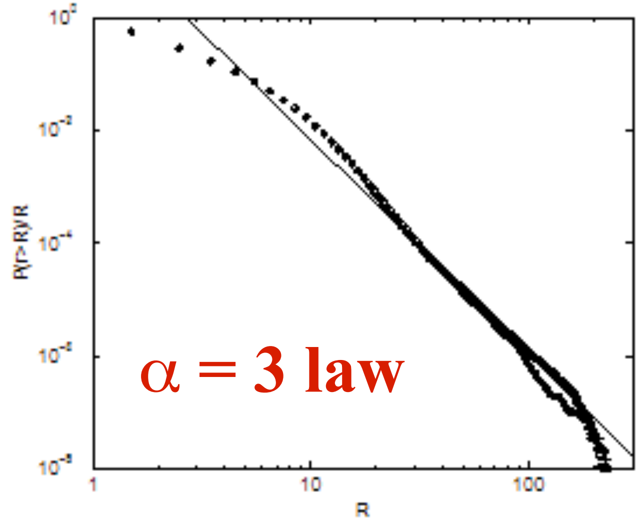
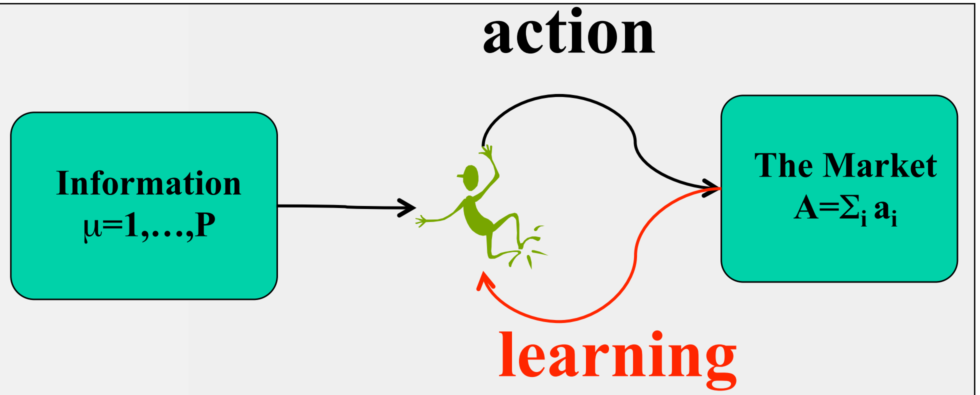


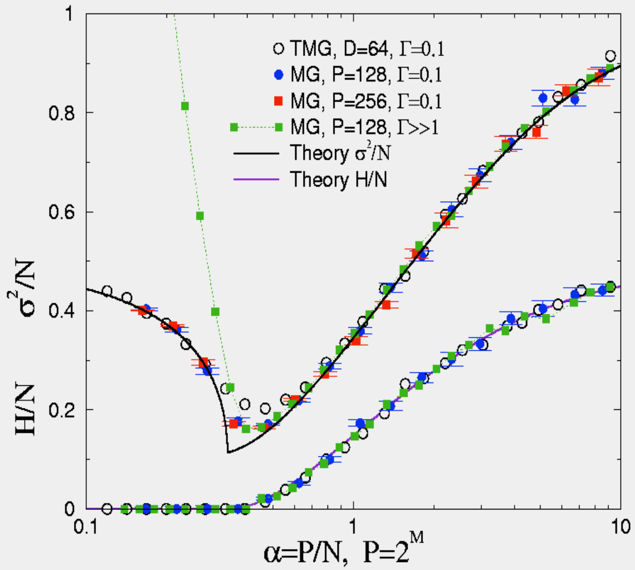
FIG. 7. Cumulative function of the returns R divided by the return R (circles: positive returns, x: negative returns) ($P = 16$, $S = 2$, $N_s = 1001$, $N_p = 1200$, $\epsilon = 0.01$); the continuous line has a slope of −3.8, close to the one observed in financial markets.

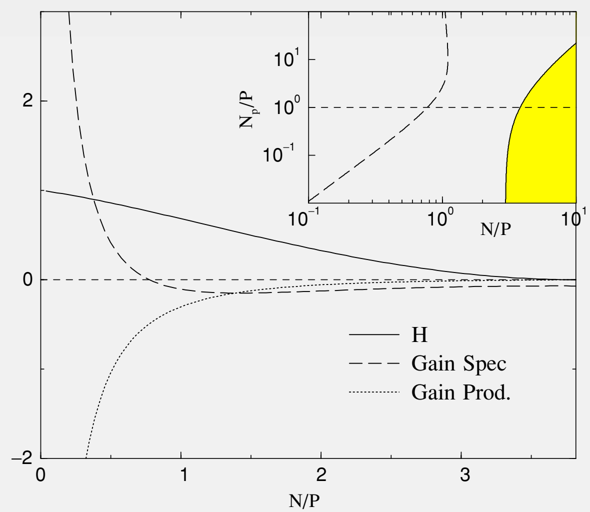
Large fluctuations when market aggregates efficiently information

Non trivial issues

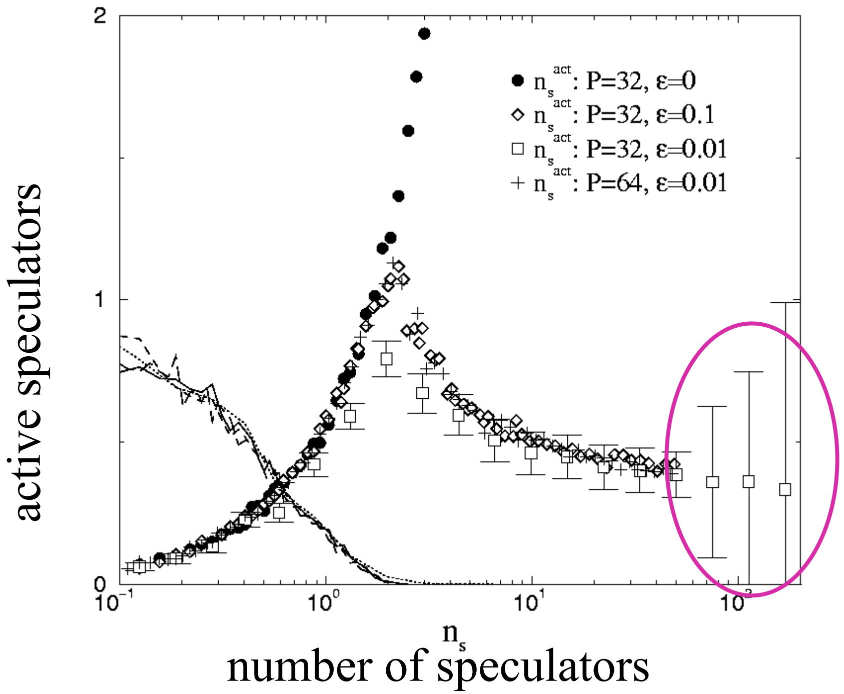


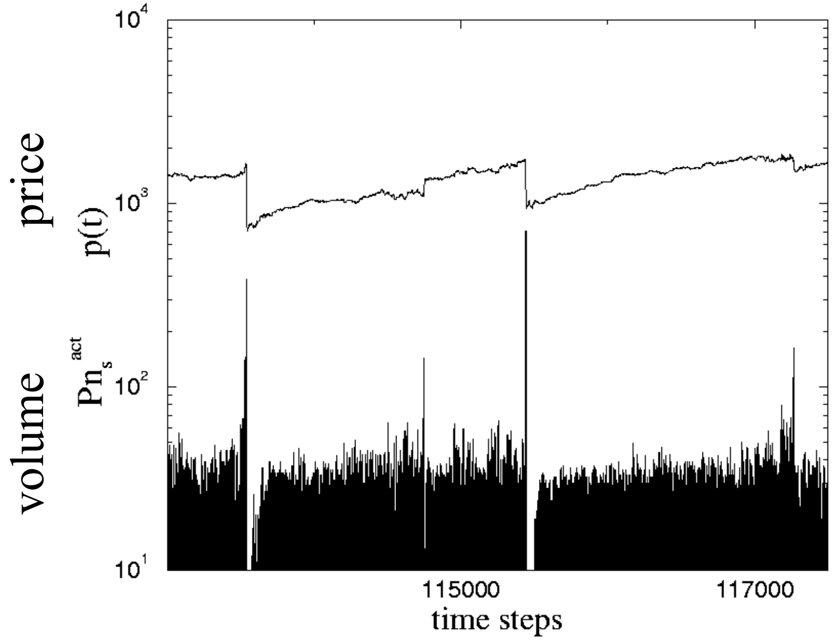
- Dependence on initial conditions
- Irrelevance of the origin of information
- Independence on "temperature"
- Noise ~ 1/fluctuations (cfr <v^2>=KT)
- Market ecology
- Market crashes
- Instability with finite memory
- Tobin tax reduces volatility
- Market impact of transactions
- ...

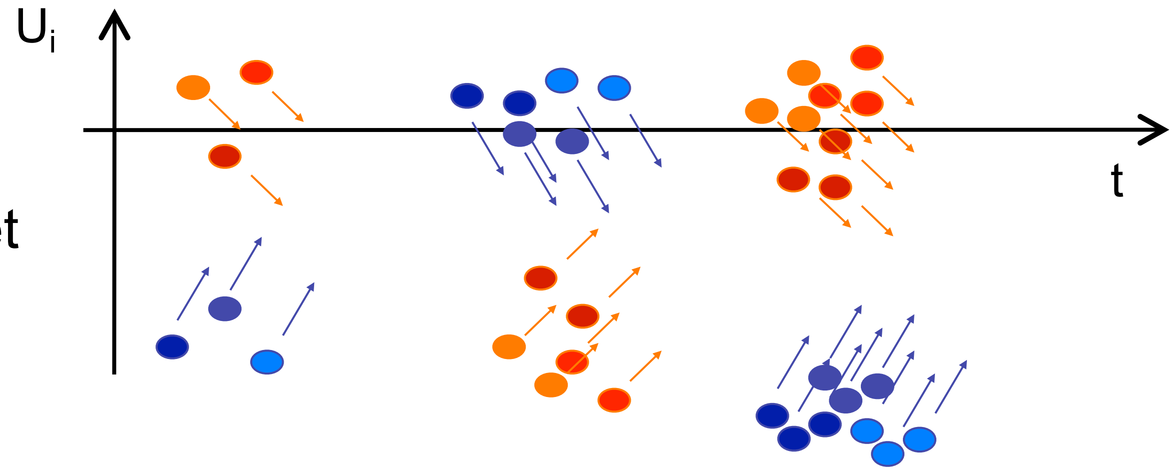




Market crashes occur precisely when there are too many types of traders







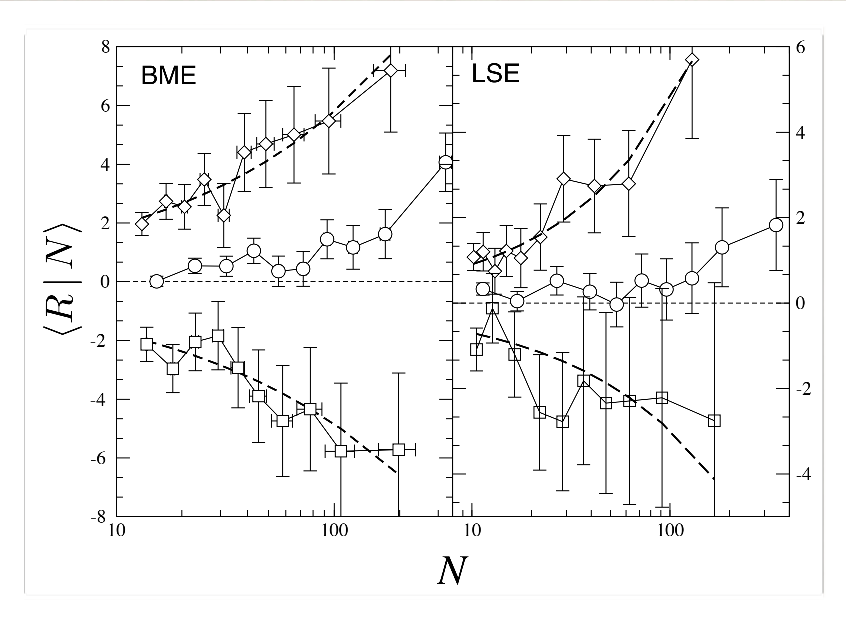
Speculators outside the market do not “see” each other!

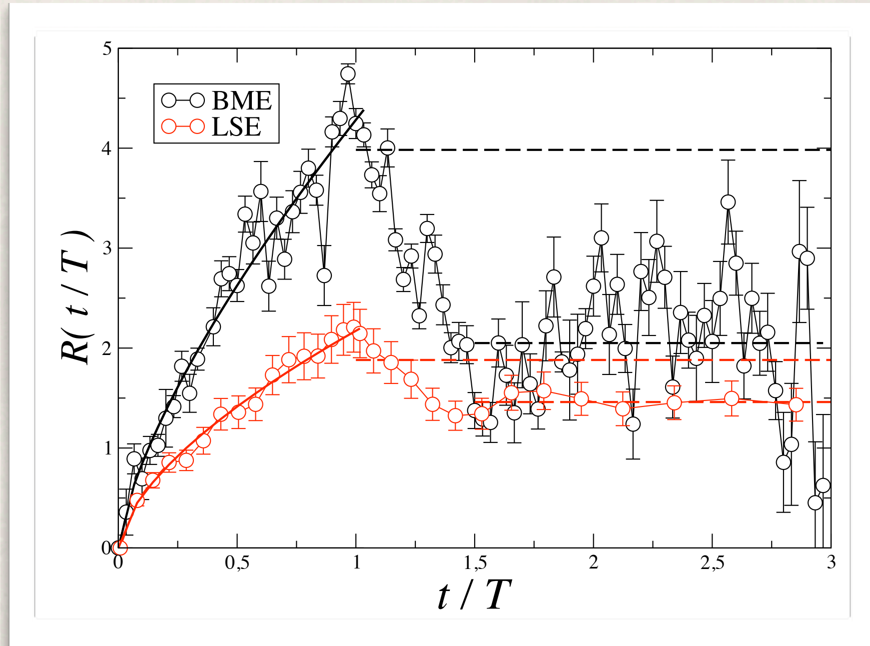
Market impact of meta-orders

Meta-orders

- Markets operate in a regime of vanishing revealed liquidity, but large latent liquidity.
- The market is not liquid enough for the execution of large orders,
- In order to limit execution costs liquidity takers need to split large order into many child orders that are executed sequentially
- In order not to reveal their strategies meta-orders need to be executed over a long period of time, injecting predictable patterns that are exploited by liquidity providers
- Strategic interaction between liquidity providers and liquidity takers
  - Representative agent models (e.g. Kyle '85)
  - Phenomenological models (e.g. Bouchaud et al. '08, ...)

Properties of Meta-orders





Moro *et al.*, Phys. Rev. E 80, 066102 (2009)

$$\Delta(Q) = Y\sigma\sqrt{\frac{Q}{V}}$$

Tóth et al. Phys Rev. X, 2011, 1 021006 (2011)

Q = metaorder size

Δ = relative price change = $R(1)$

V = daily volume

σ = daily volatility $\quad Y \approx 1$

Grand-Canonical Minority Game

The excess demand measures the unbalance between buy and sell orders:

$$A(t) = \sum_{i=1}^{N_p} a_{i,\mu(t)}^{(P)} + \sum_{i=1}^{N_s} a_{i,\mu(t)}^{(S)} \phi_i(t)$$

Liquidity providers update their score:

benchmark

$$U_i(t+1) - U_i(t) = -a_{i,\mu(t)}^{(S)} A(t) - \epsilon_i$$

and decide if playing or not accordingly:

$$\phi_i(t) = \begin{cases} 1 & \text{if} \quad U_i(t) > 0 \\ 0 & \text{if} \quad U_i(t) < 0 \end{cases}$$

Price is given by:

$$\log p(t+1) - \log p(t) = \frac{1}{P} A(t)$$

Meta-orders in Minority Games

Modeling a uniform meta-order of size Q=hT starting at $t = 0$ and ending at $t = T$ by adding a fixed buyer:

$$\Rightarrow \qquad A(t) = A_{h=0}(t) \rightarrow A_h(t)$$

With the assumed market clearing condition market impact is:

$$\Delta(t) = \frac{1}{P} \sum_{0 \leq s < t} \overline{\left\langle A_h(s) - A_{h=0}(s) \right\rangle}$$

The most efficient way of taking care of the $A_{h=0}(s)$ contribution is by simulating virtual markets.

GCMG and Market Impact

The meaningful quantities are:

- permanent impact $$\Delta^* \equiv \lim_{t\to\infty} \Delta(t)$$
- average execution cost $$p(0)\overline{\Delta} = \frac{p(0)}{T}\sum_{t=0}^{T}\Delta(t)$$
- normalized execution cost $$\Delta(t) \simeq t^\alpha \Rightarrow \frac{\bar{\Delta}}{\Delta(T)} \simeq \frac{1}{1+\alpha}$$