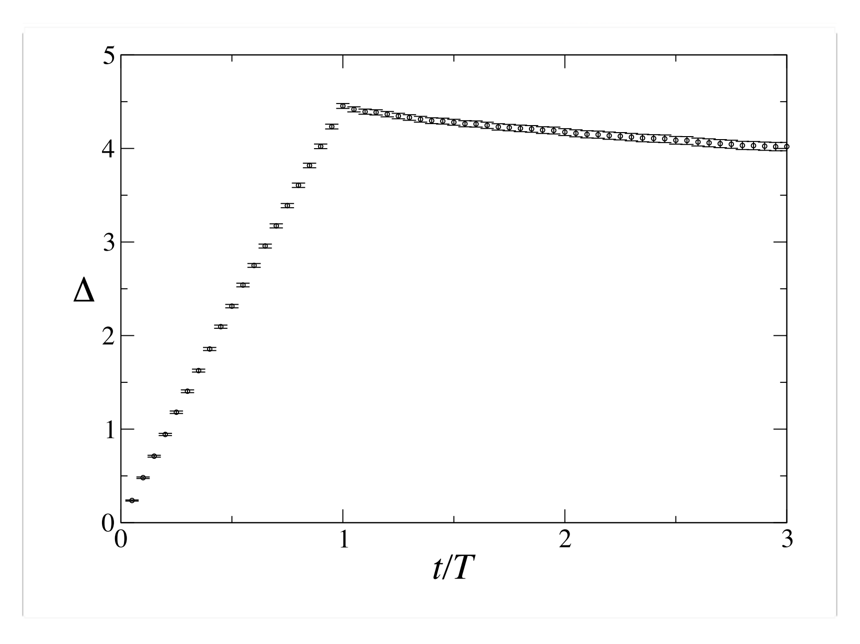
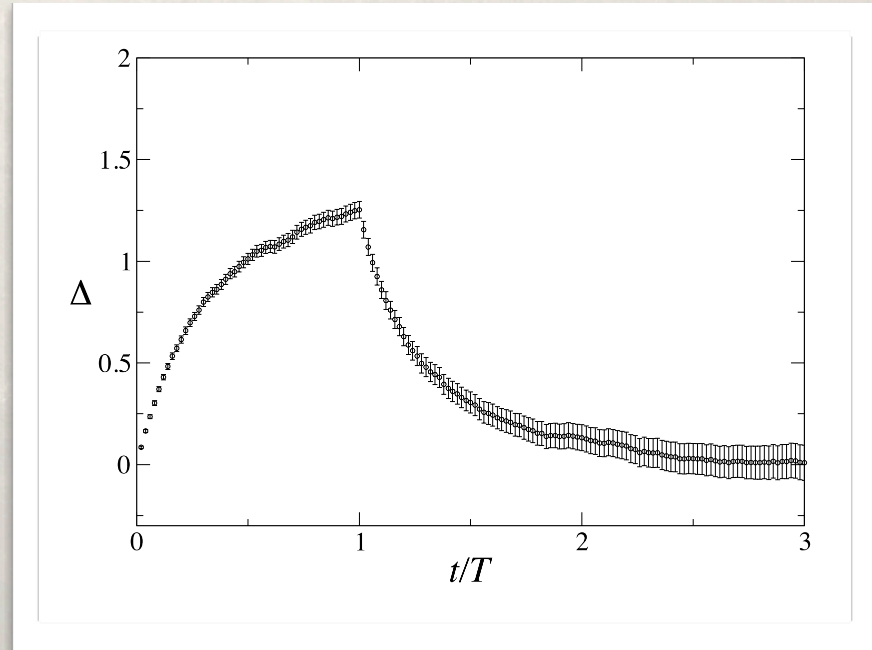
Impact and Predictability

In the predictable phase ($H > 0$) the impact is linear and the permanent impact is non-zero.

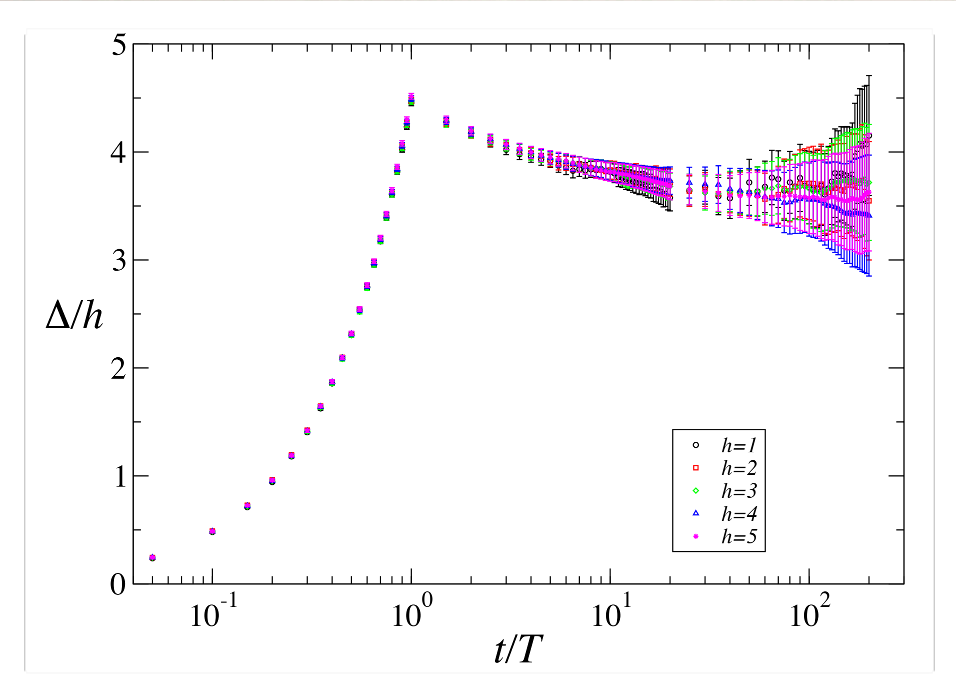


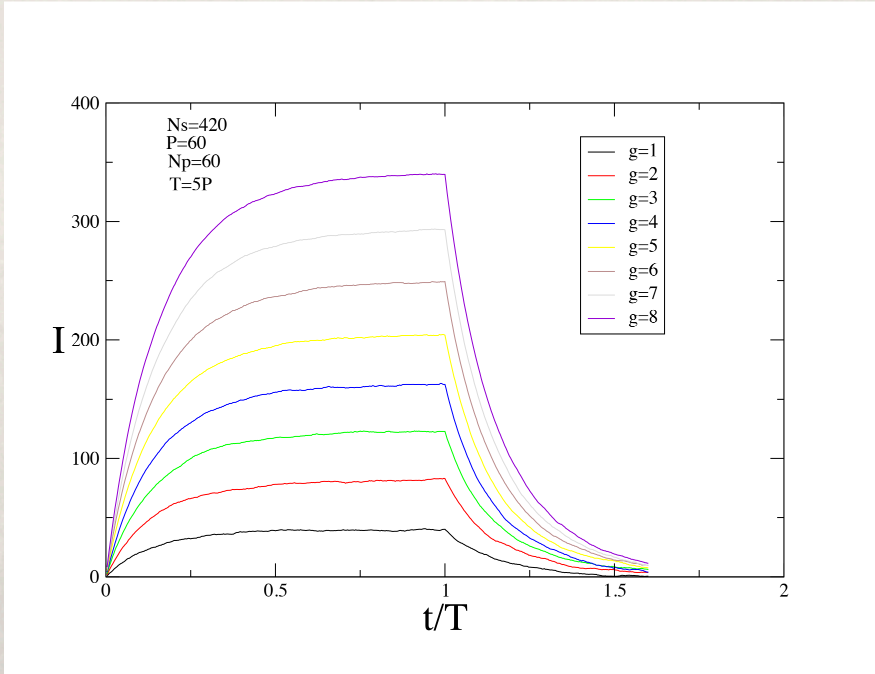
In the unpredictable phase ($H = 0$) the impact is concave and the permanent impact is zero.



Impact and order size

The impact scales linearly with the perturbation h:





Impact from linear response theory

The partition function for the perturbed Hamiltonian is:

$$Z_h = \mathrm{Tr}_{\mathbf{m}} \exp\left(-\frac{\beta}{2N_s}\sum_{\mu}\left(\langle A|\mu\rangle + h\right)^2\right)$$

we have:

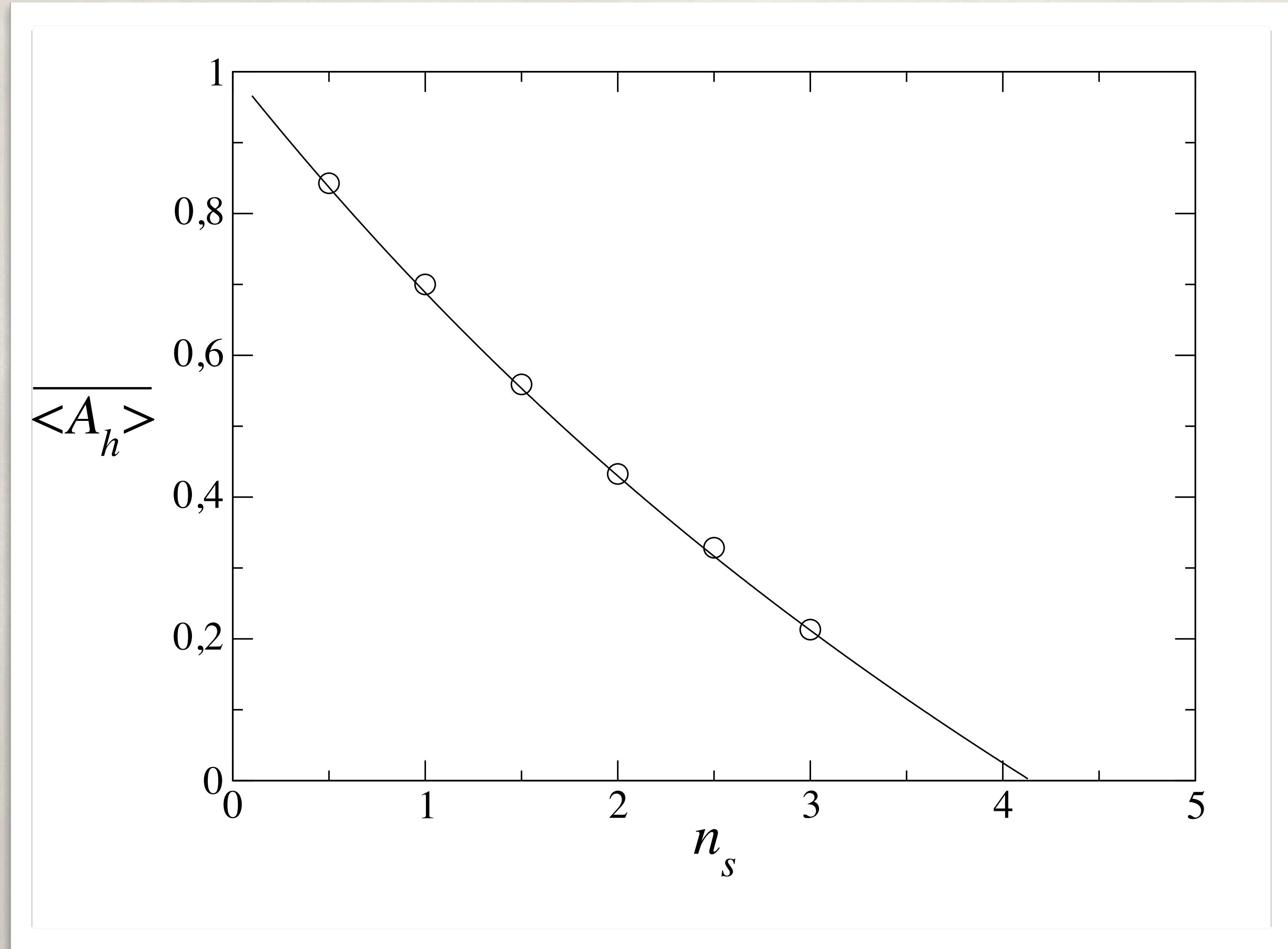
$$\lim_{N_s\to\infty}\lim_{\beta\to\infty}\left(-\frac{N_s}{\beta P}\right)\left(\frac{d\ln Z_h}{dh}\right) = \left\langle \frac{1}{P}\sum_{\mu}\left(\langle A|\mu\rangle + h\right)\right\rangle_{\mathbf{m}} = \langle A_h\rangle$$

averaging over the random strategies of agents:

$$\overline{\langle A_h\rangle} = \frac{N_s^2}{P}\frac{d}{dh}f_h = \frac{h}{1+\chi} \qquad f_h(G,g,R,\rho) = \frac{h^2 P}{2N_s^2(1+\chi)} + f(G,g,R,\rho)$$

and χ is the susceptibility of the unperturbed system.

Impact and Predictability



Permanent Impact

Reformulating the dynamics in continuum time, taking into account the fact that the characteristic times of the GCMG are of order P:

$$\tilde{h}(\tau) = h(t) \qquad \tau = \frac{t}{P}$$

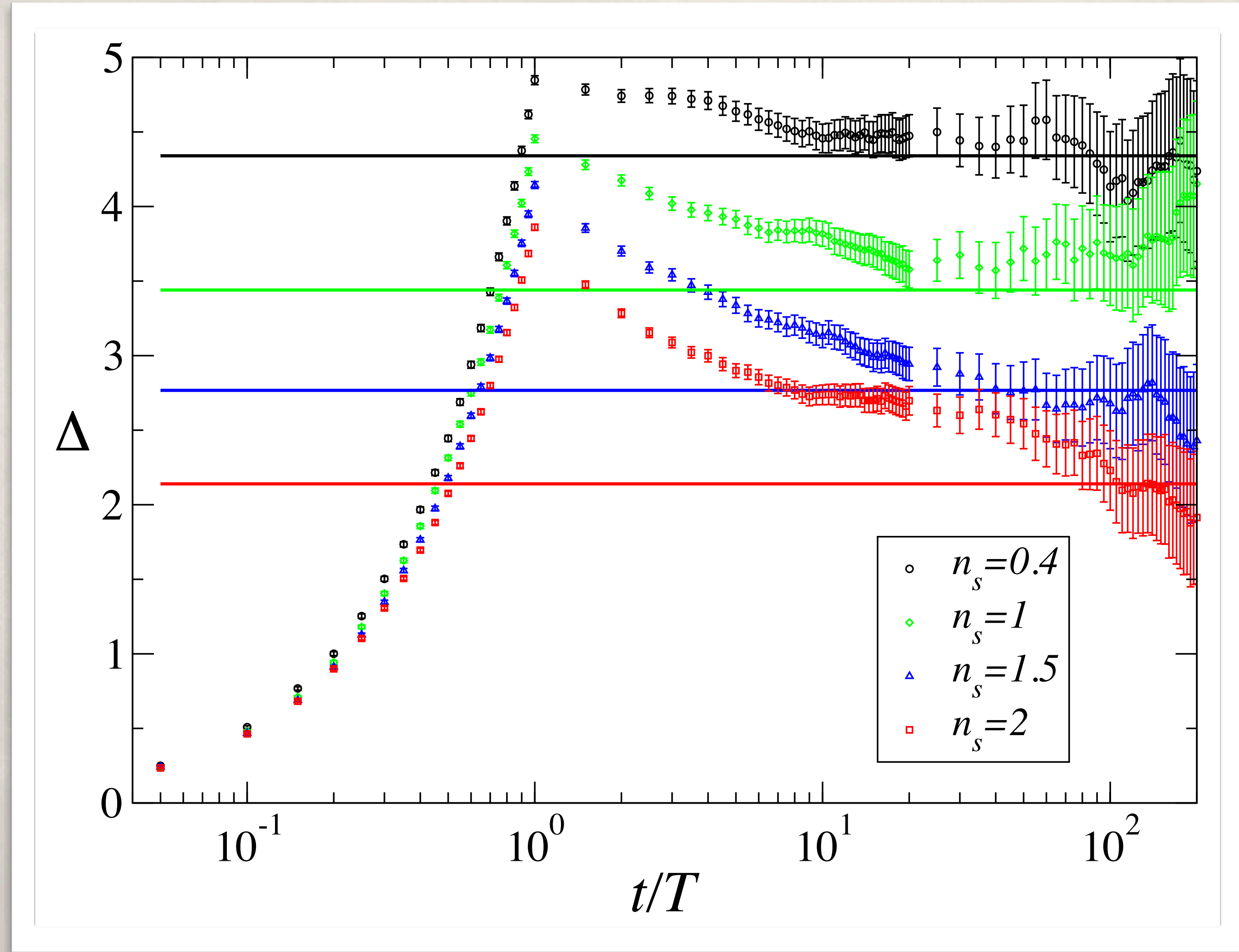
As long as the perturbation h is smaller than $A \simeq P^{1/2}$ linear response theory applies (the kernel relaxes faster than a power-law, as expected far from criticality):

$$\overline{\langle A_h(t) \rangle} = \overline{\langle A_{h=0}(t) \rangle} + \frac{1}{1+\chi} \int_{-\infty}^{t/P} d\tau \; K(t/P - \tau)\tilde{h}(\tau)$$

And it is possible to calculate the permanent impact:

$$\Delta^* = \frac{hT}{P(1+\chi)}$$

Permanent Impact



Execution costs

$$\frac{\bar{\Delta}}{\Delta(T)} = 1 - \frac{1}{2}\left(\frac{\kappa_T^0 - \kappa_T^2}{\kappa_T^0 - \kappa_T^1}\right) \qquad \kappa_T^m = \int_0^1 dx\, x^m K(xT/P)$$

If τ_c is the relaxation time of the kernel:

$$\frac{\overline{\Delta}}{\Delta(T)} \approx \begin{cases} \frac{1}{2} + \frac{T}{12P}\frac{\chi K_r(0)}{1+\chi} & \text{for} \quad T/P \ll \tau_c \\ \frac{1}{2} & \text{for} \quad T/P \gg \tau_c \end{cases}$$

To be compared with the empirical value $0.6 \div 0.7$.

Farmer, Gerig, Lillo, Waelbroeck, arXiv 1102.5457 (2011)

Conclusion I

- Minority games as statistical mechanics of financial markets

- Impact and information efficiency:
  Non-zero permanent (linear) impact for H>0
  Zero permanent (concave) impact for H=0
  Average execution cost > (permanent impact)/2
  only for T<<relaxation time

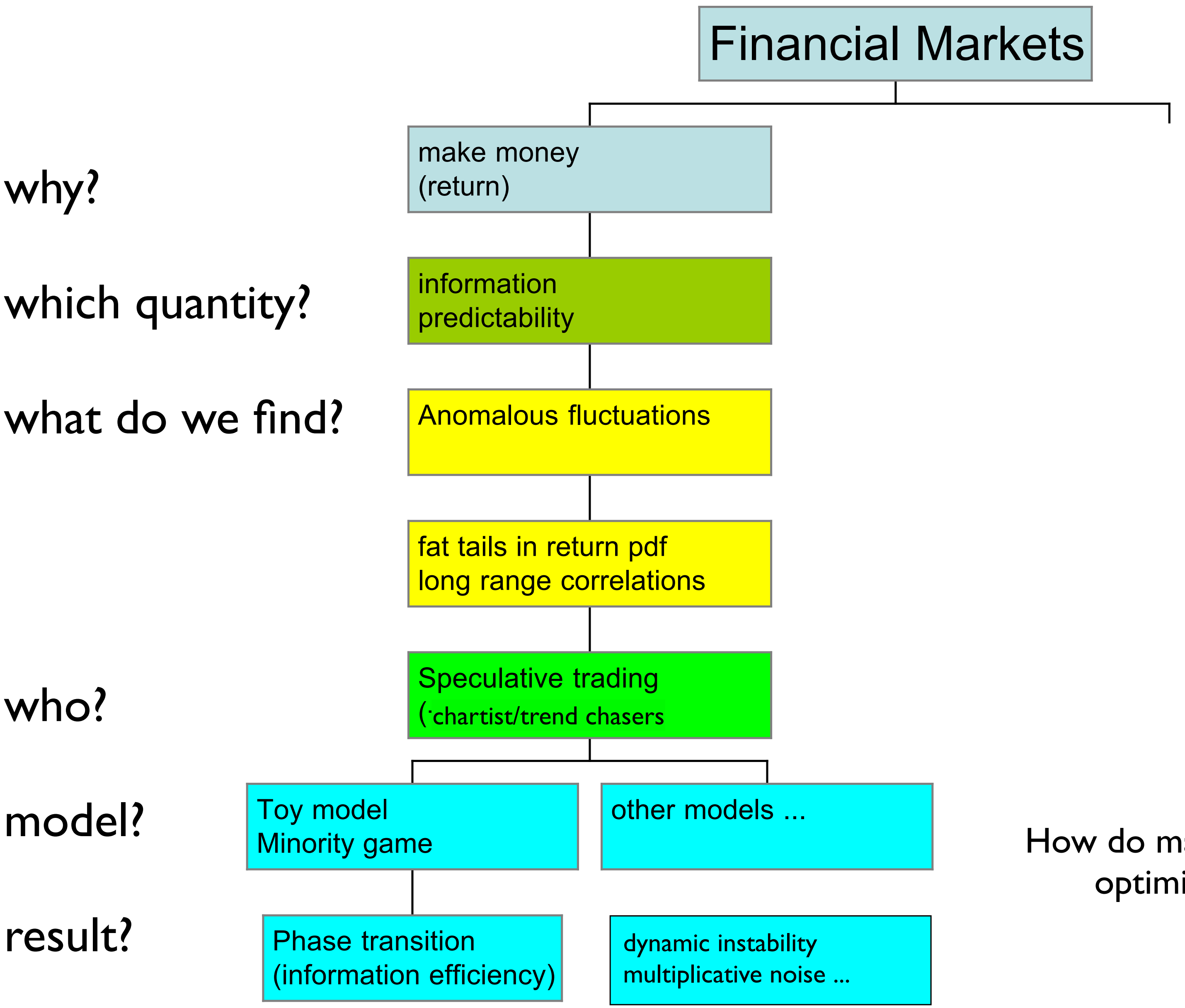
- Beyond linear impact: V-shaped latent order book (Tóth et al. 2011)

$$Q = \int_{p(0)}^{p(0)+\Delta} dp \mathcal{V}(p) \propto \Delta^2$$

- Towards a quantitative theory of impact:
  fitting Minority Games on real market data

Managing financial risk

A personal brief overview:



How do markets respond to risk optimization strategies?

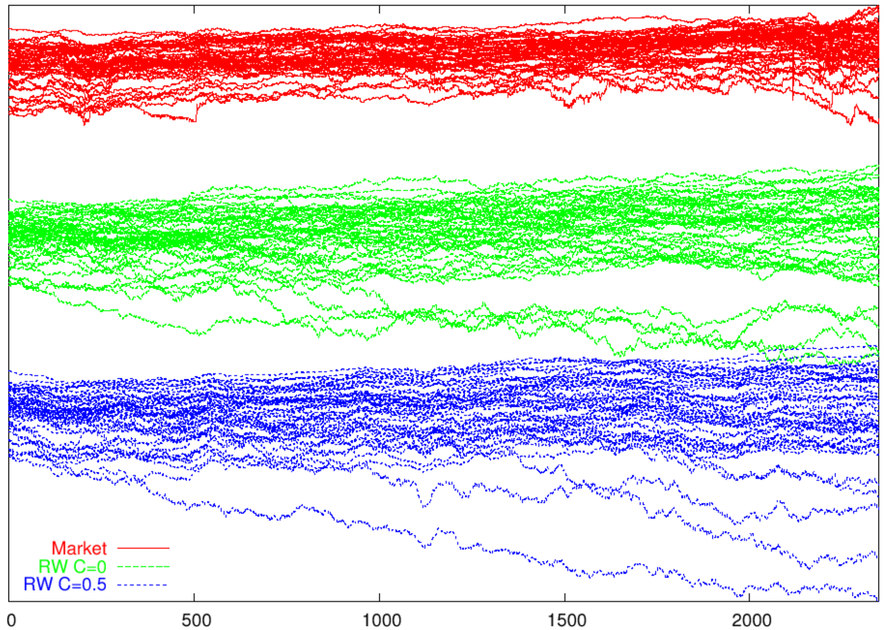
Markets respond to speculation becoming less predictable

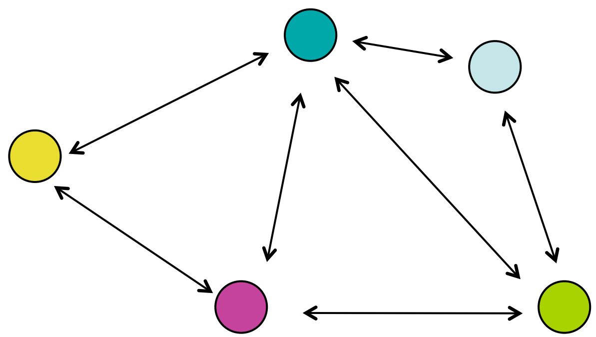
Multi-asset markets as many "particle" interacting systems

$$x_i(t) = \log p_i(t)$$

$$i = 1, \ldots, N$$
$$(N = \# \text{ assets})$$

- Correlation vs interaction



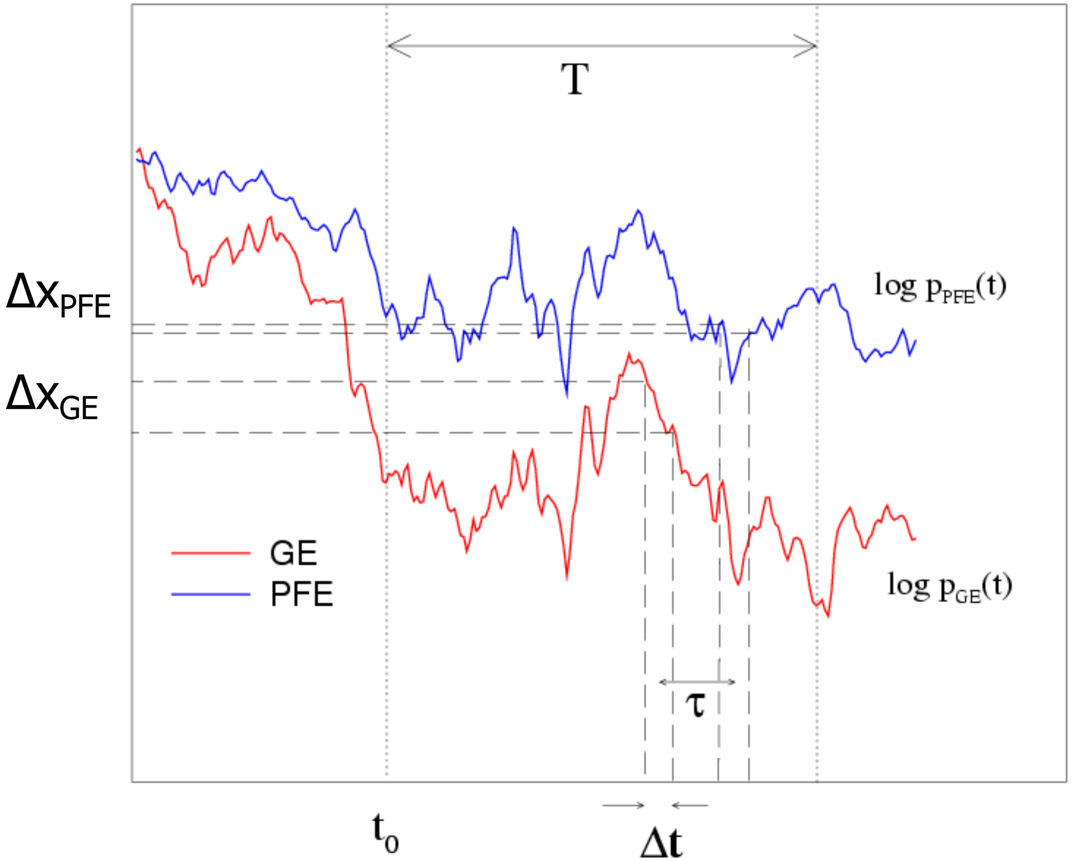


Prices move due to trading

What "moves" traders?

Probing N-particle dynamics: correlation between assets

$$\mathrm{Cov}_{\mathrm{GE,PFE}} = E[\Delta x_{\mathrm{GE}} \Delta x_{\mathrm{PFE}}]$$

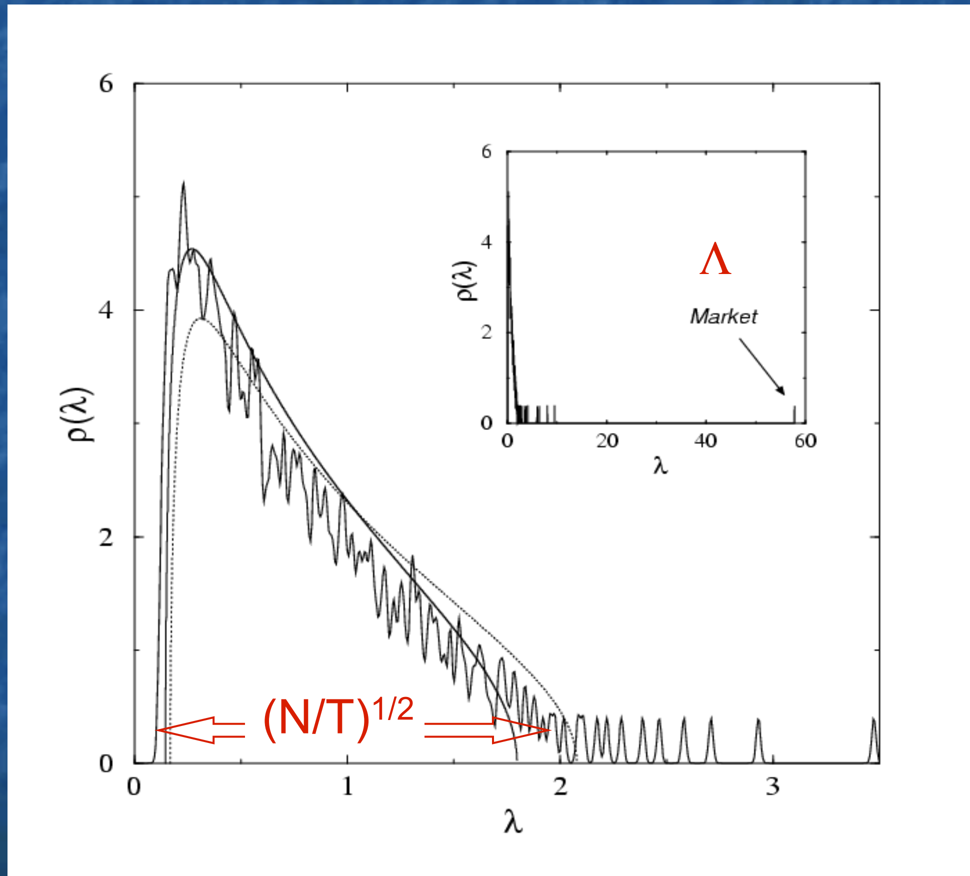


T = window size
t_0 = initial time
Δt = time scale
τ = time shift

Noise or real correlations?

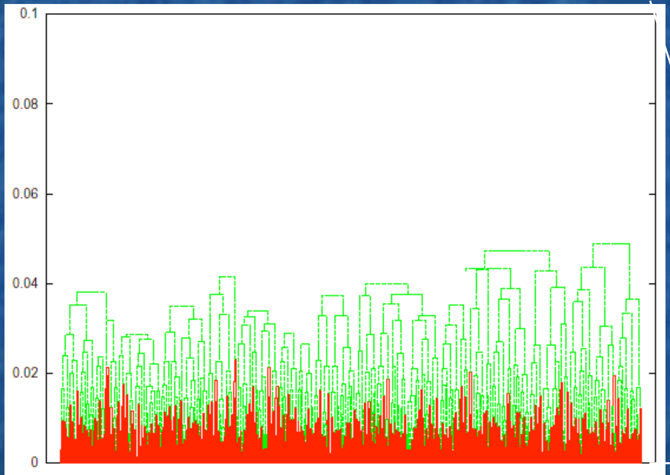
- Eigenvalue distribution and random matrix theory
  (Laloux et al./Gopikrishnan et al. …)

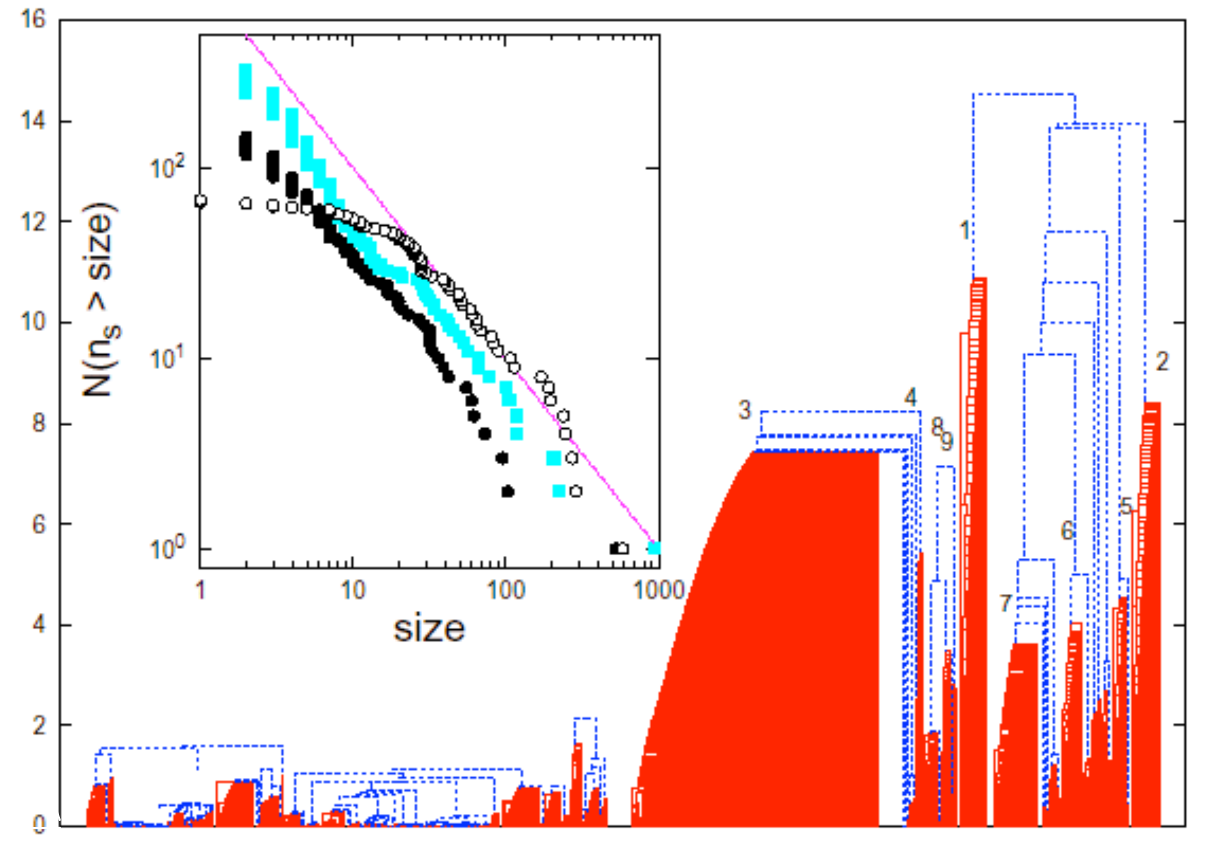
- The bulk of eigenvalue distribution is dominated by sampling effects (noise)
- One large eigenvalue (market mode)
- Few eigenvalues with "economic" meaning
- "Localized" small eigenvalues



Hierarchical clustering of assets (N=2000 NYSE 90-98):

- Statistically significant
- Zipf's law
- no orthogonality





Data from R.N. Mantegna

Note: totally unsupervised method
Number of clusters not predefined!

Translation symmetry

Economic forces invariant under:

$$p_i \to \lambda p_i \; \forall \text{assets } i, \quad \text{wealth} \to \lambda \text{ wealth}$$

i.e.:

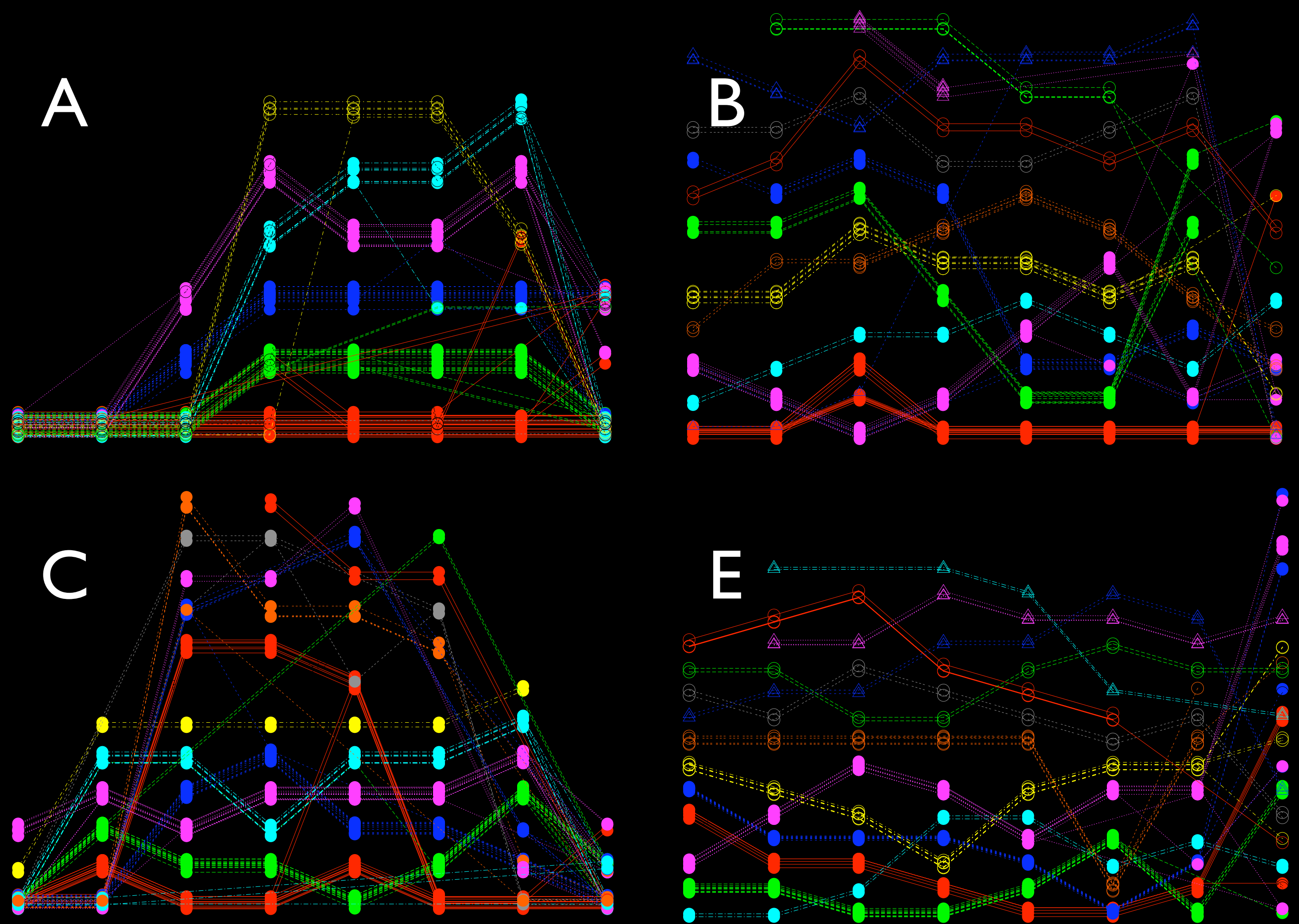
$$x_i = \log p_i \to x_i + x_0 \; \forall \text{assets } i$$

In physics:

Degrees of freedom
- center of mass (external forces) $$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$
- relative coordinates (internal forces) $$x_i - \bar{x}$$

→ remove the center of mass dynamics from correlations

Time-horizon invariant structure without center of mass



Assets in the same cluster follow the same trajectory in sets B, D, E

Very different structure in overnight returns

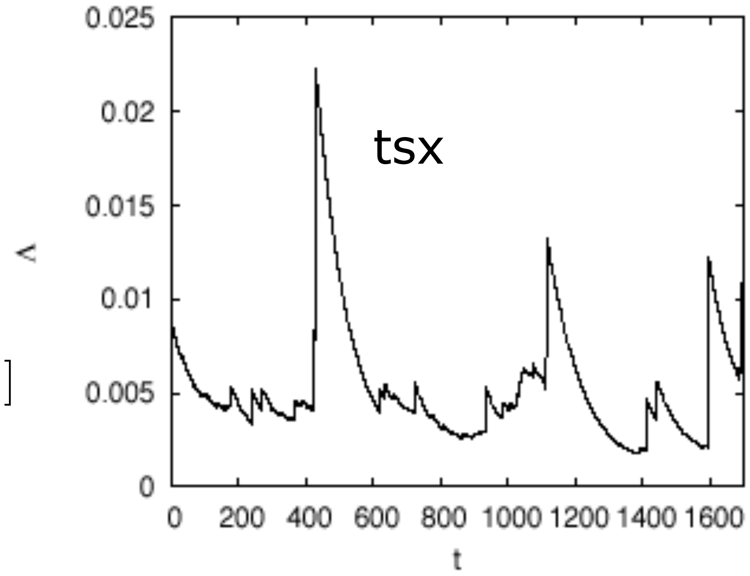
The dynamics of the center of mass

- empirical findings

- phenomenological model

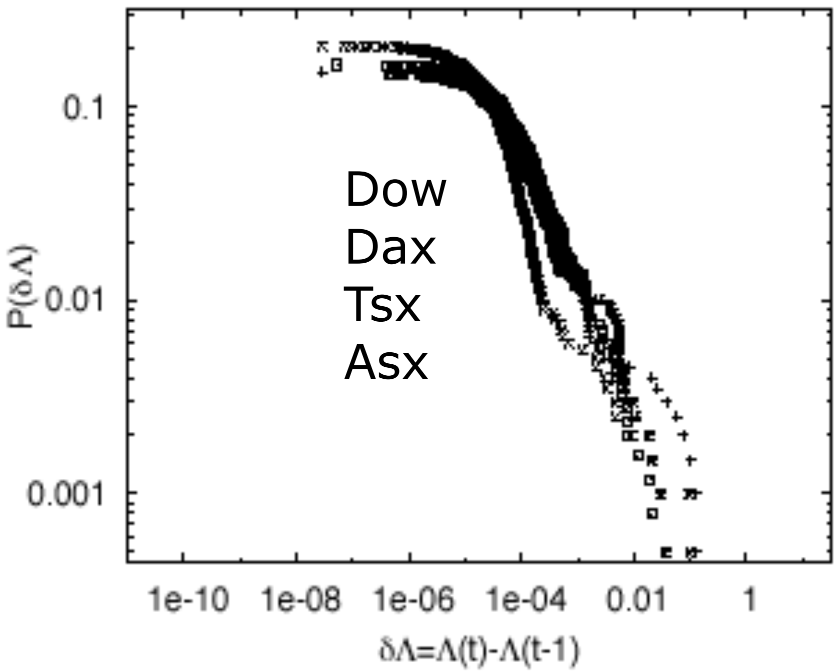
G. Raffaelli, MM JSTAT L08001 (2006)

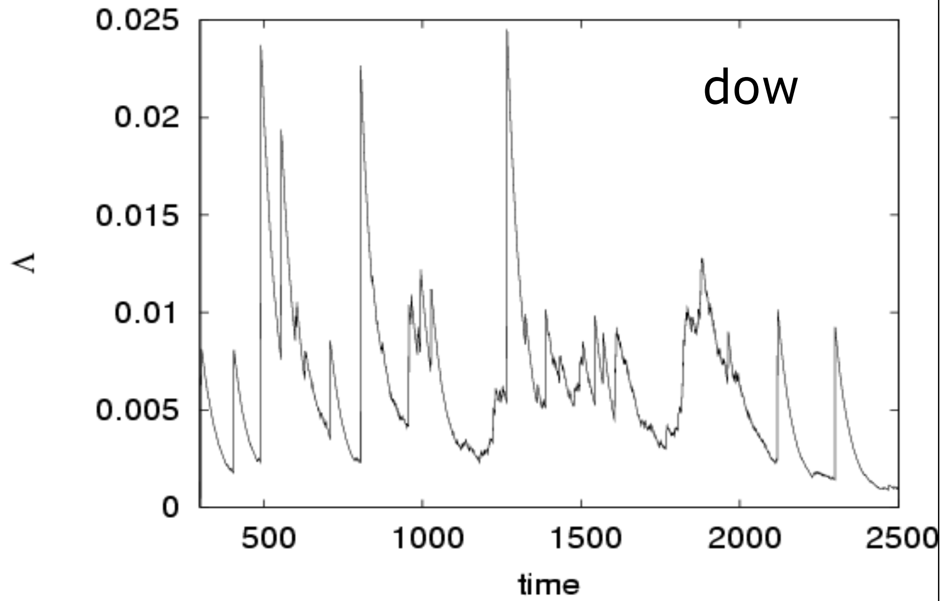
The dynamics of the largest eigenvalue of the covariance matrix



$$\mathrm{Cov}_{i,j}(t) = \mu \sum_{t' \leq t} (1-\mu)^{t-t'} [x_i(t') - r_i(t)][x_j(t') - r_j(t)]$$

$$r_i(t) = \mu \sum_{t' \leq t} (1-\mu)^{t-t'} x_i(t')$$





(see also Drozdz et al.)

Where do correlations come from?

- Portfolio investment:
  agents spread investment across stocks to minimize risk
  (i.e. avoid correlations)
  In doing this, they invest in a correlated way in the market →
  they create correlations

$$\hat{C} = \hat{B} + \hat{F}\left(\hat{C}\right) + \hat{\Omega}$$

- Simple model describing this feedback
  (MM, Ponsot, Raffaelli JEDC 2009)

Optimal Portfolio

- Problem: Invest $\mathbf{z}=(z_1,\dots,z_N) \rightarrow$ stochastic return = $\boldsymbol{\delta}\mathbf{x}\cdot\mathbf{z}$
  - expected return = $\mathbf{r}\cdot\mathbf{z} = r_1 z_1 + \dots + r_N z_N = R$ ($\mathbf{r}=E[\boldsymbol{\delta}\mathbf{x}]$)
  - wealth = $\mathbf{1}\cdot\mathbf{z} = z_1 + \dots + z_N = W$

  so as to minimize risk $\Sigma(\mathbf{z})$
- Solution (if $\Sigma(\mathbf{z})=\mathrm{Var}(\mathbf{z})$):

$$\mathbf{z}^* = \arg\max_{\mathbf{z},\lambda,\nu} \left[\frac{1}{2}\mathbf{z}'\hat{C}\mathbf{z} + \lambda(R - \mathbf{z}\cdot\mathbf{r}) + \lambda(W - \mathbf{z}\cdot\mathbf{1}) \right]$$

- Note:

  no impact on market.

  unique solution. Many traders can invest in the same way.

  Will this have some impact?

The simplest model:

Closed dynamical model, self-generated fluctuations/correlations

- $x_i(t+1) = x_i(t) + b_i + \eta_i(t) + [\varepsilon+\xi(t)]\, z_i(t)$

  b_i = "bare" return of asset i

  $\eta_i(t)$ = "bare" noise $E[\eta_i(t)\, \eta_j(t)] = B_{i,j}$ bare correlation

  $\varepsilon+\xi(t)$ = portfolio investment rate $E[\xi(t)^2]=\Delta$

- Where

$$\mathbf{z}^* = \arg\min_{\mathbf{z},\lambda,\nu}\left[\frac{1}{2}\mathbf{z}\times C\times\mathbf{z} - \lambda\left(\mathbf{z}\times\mathbf{r} - R\right) - \nu\left(\mathbf{z}\times\mathbf{1} - W\right)\right]$$

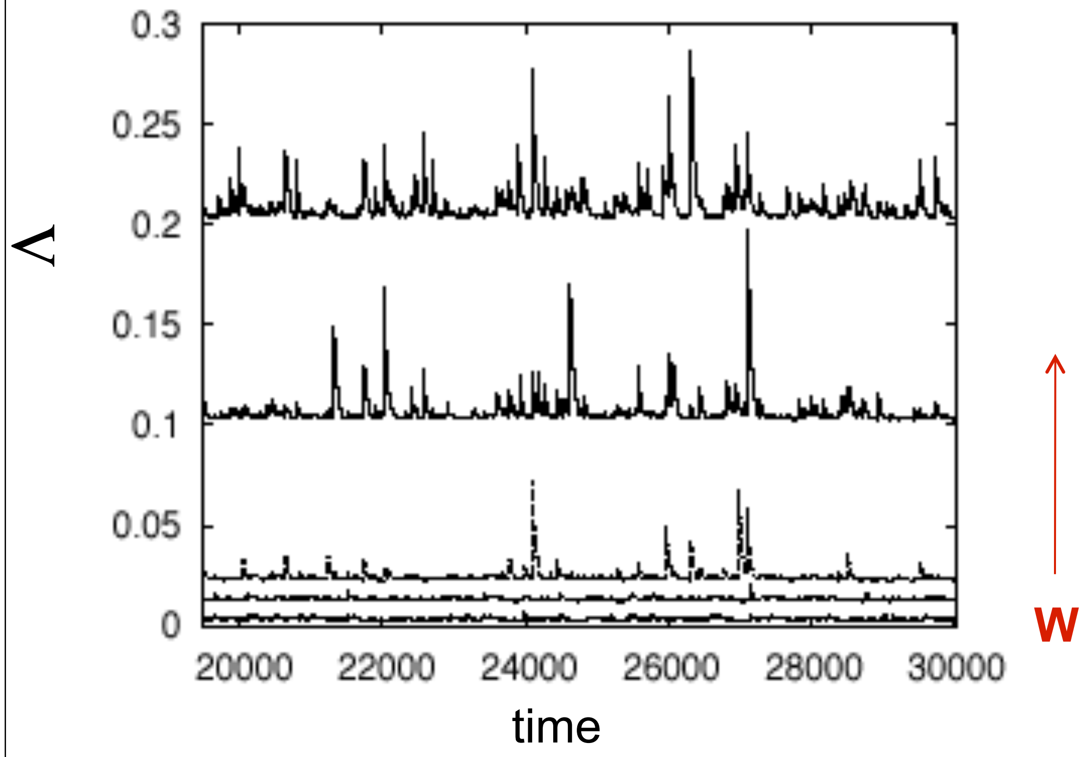
- Average return and correlation matrix ($\mu \sim 1/T_{average}$)

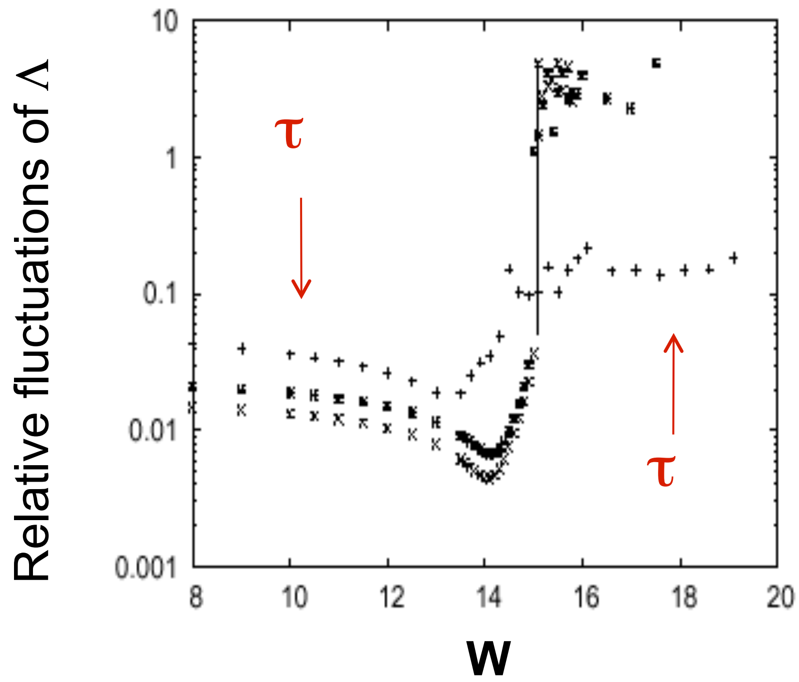
  $r_i(t+1) = (1-\mu)\, r_i(t) + \mu\, [x_i(t) - x_i(t-1)]$

  $C_{i,j}(t+1) = (1-\mu)\, C_{i,j}(t) + \mu\, \delta x_i(t)\, \delta x_j(t)$

  $\delta x_i(t) = x_i(t) - x_i(t-1) - r_i(t)$

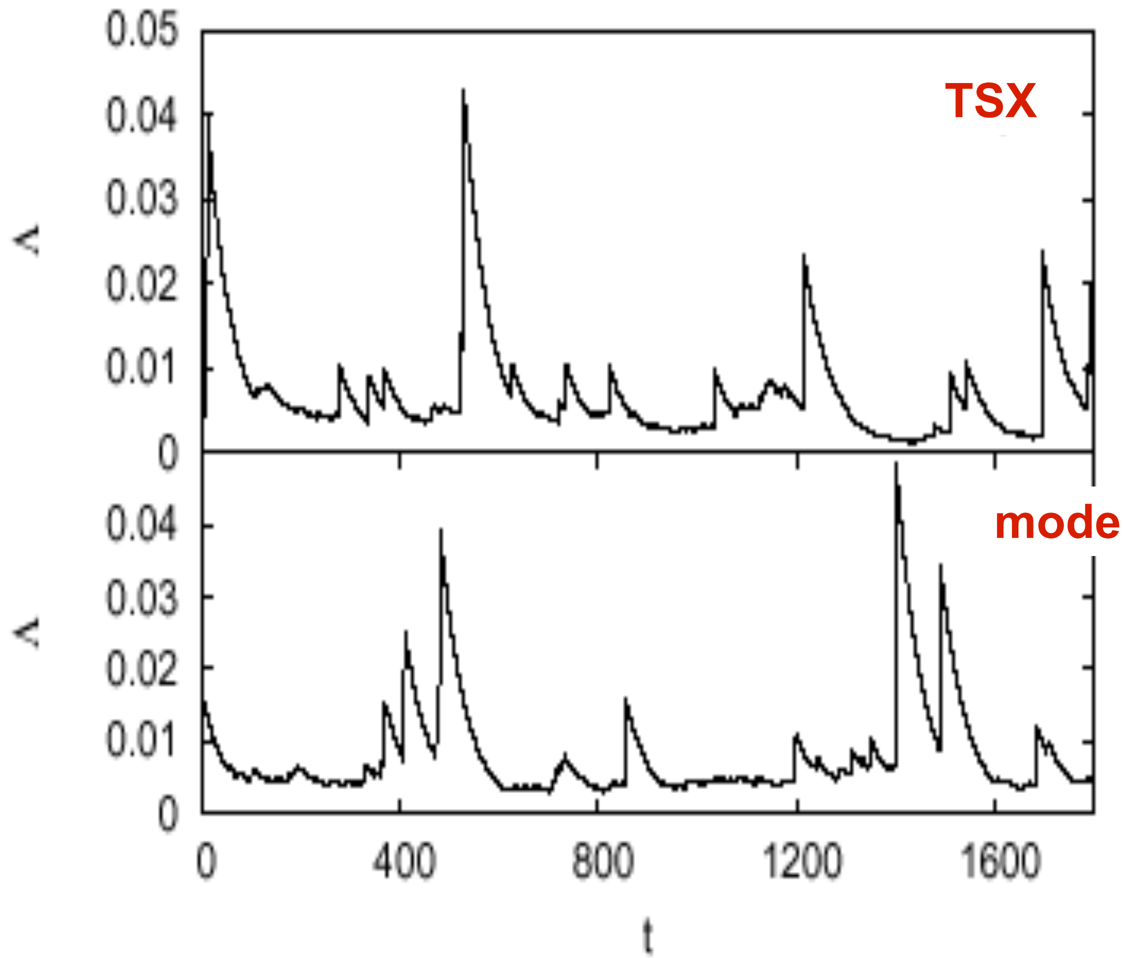
Numerical simulations

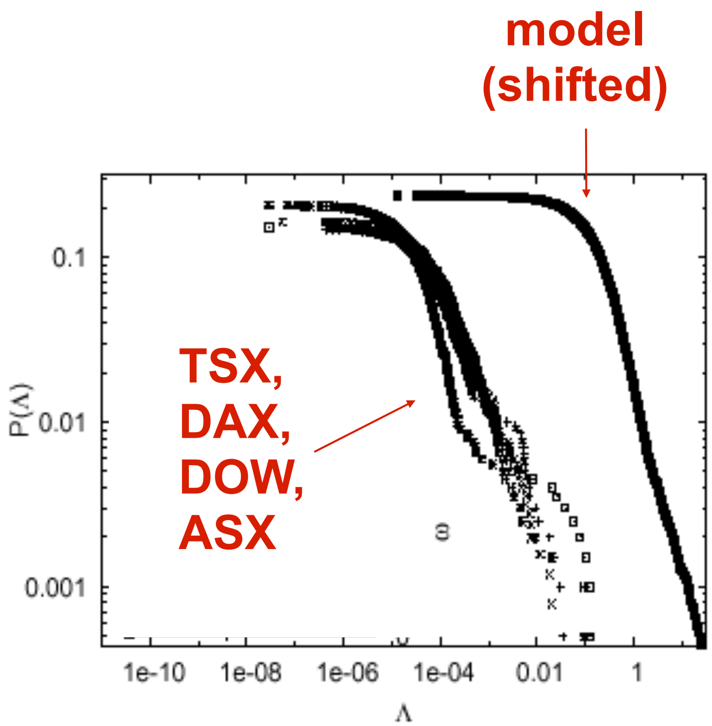


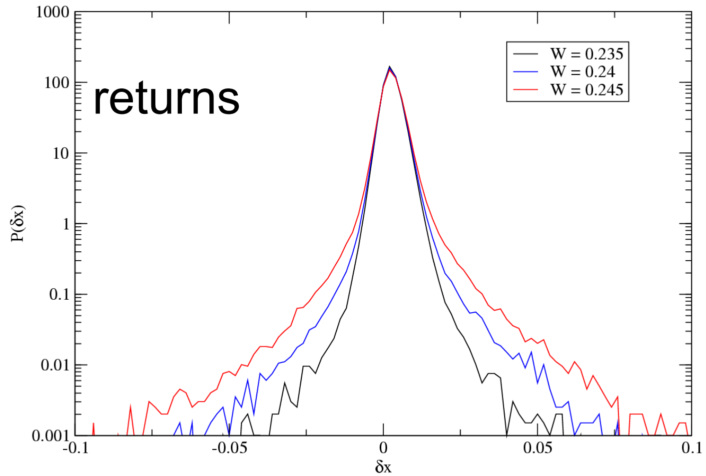


Dynamic instability as W→W*

…and close to W*







Theory: low frequency limit $\mu \rightarrow 0$

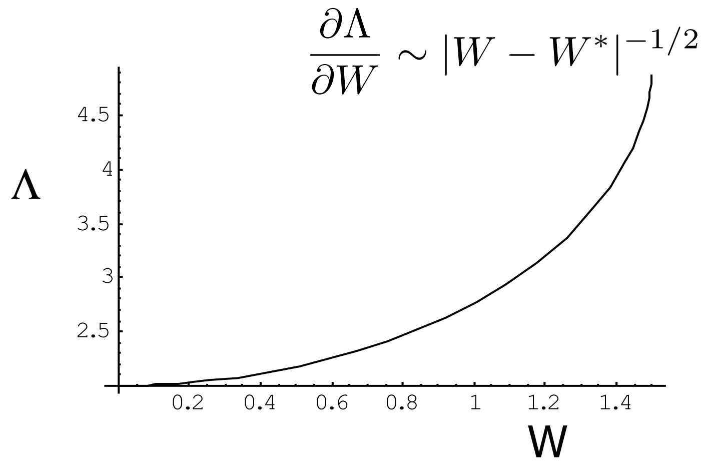
- low frequency limit $\mu \rightarrow 0$

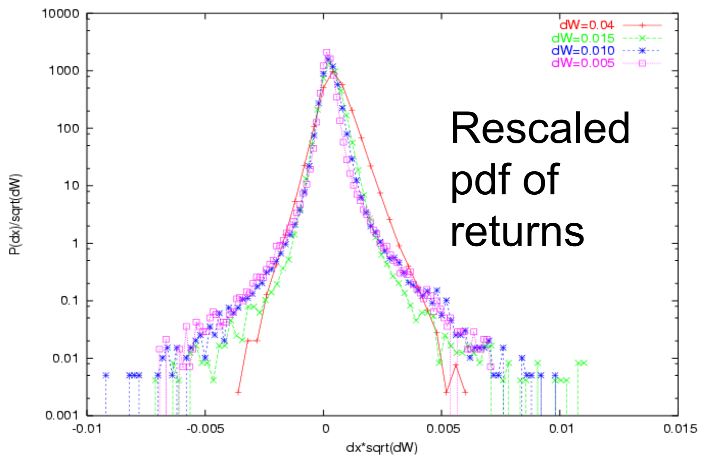
  C(t), $r_i(t)$ independent of t → $z_i(t)$ independent of t, $\delta z_i(t)=0$

- Self-consistent equations

  → phase transition at W* to dynamically unstable phase

- Small frequency expansion scaling $$\frac{\delta\Lambda}{\Lambda} \sim \sqrt{\frac{\mu}{W^* - W}}$$

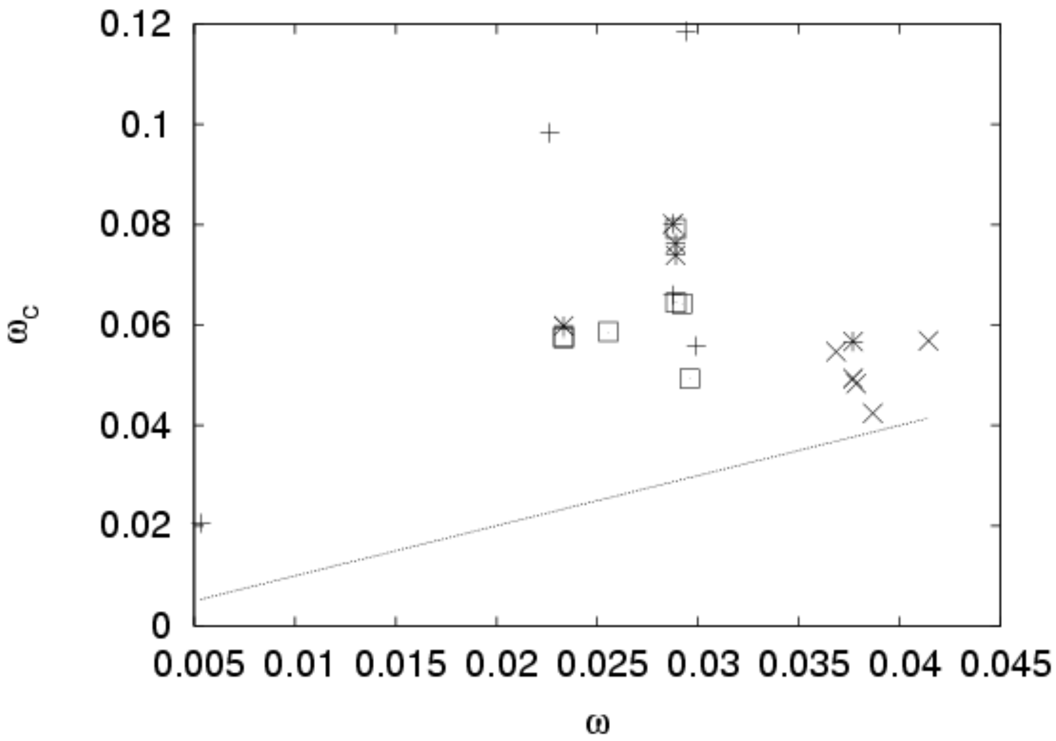




General results:

- the phase transition is quite robust
  - for any choice of risk measures of agents
  - independent of the presence of higher order derivative terms
- the market is less stable
  - the larger the volume of trading W
  - the smaller the return demanded R
  - the larger “bare” returns (~ dividends)
  - the more correlated are stocks a priori
  - if there is a lot of trading in trend following strategies

Where are real markets?
maximum likelihood estimates



DAX (+), DOW (□), TSX (*), ASX (x)

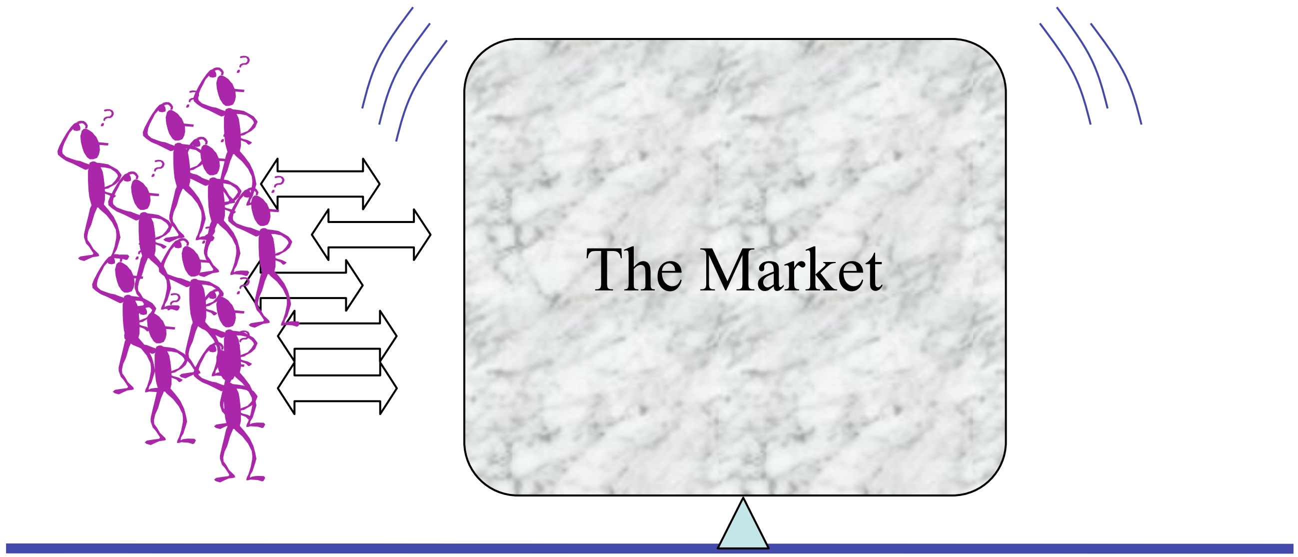
Conclusions

- log p translation invariance suggests different dynamics of center of mass and relative coordinates
- Scale invariant structure of correlations of relative coordinates:
  similar to inertial range in turbulence
- Complex dynamics of global correlations
- Phenomenological model (feedback of correlations through portfolio strategies)
  suggests system is close to a phase transition

No impact assumption

(price taking behavior)

negligible traders (~1/N) wrt market



What if $\chi=\infty$?

What if all agents behave the same?

Open problems

- A wealth of results on direct problem: inverse problem?
- Plenty of data is available
- Fitting MG:
  1- conceptually similar to Hopfield models in neuroscience
  2- indicators of market information efficiency
  3- quantitative models of market impact
- Understand structure of overnight correlations
  (inference of non-stationary models)
- ...