Optimization on networks: message-passing algorithms in a resource allocation model.

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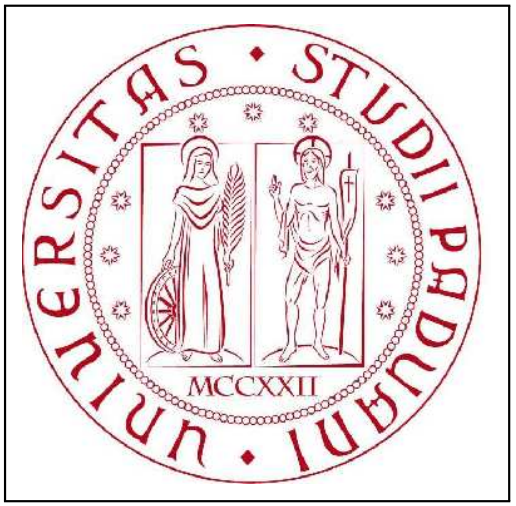
Personal background.

- **M.Sc**: 2010-12, University of Padua.
  Thesis: *Microscopic characterization of active matter*.
  Supervisors: E. Orlandini, F. Baldovin.

- **Erasmus**: 2011, Imperial College London .
  Project: *Black-Scholes and beyond*.
  Supervisor: D. Vvedensky.

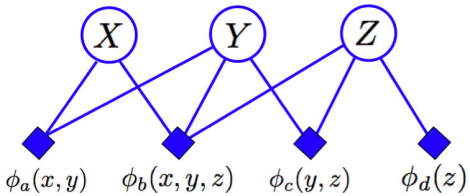
- **B.Sc**: 2007-10, University of Padua.
  Thesis: *The polydisperse brownian motion*.
  Supervisors: A. Stella, F. Baldovin.

- **Summer internship**: Summer '11, Centrica Energy, London.
  Project: *Wind performance analysis*.
  Manager: Martin Stanyon, Alejandra Sierra.



Optimization on networks: methods and key words.

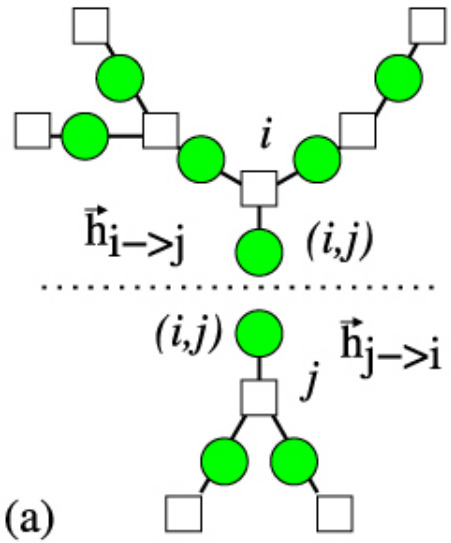
- **Model set up:** variables + cost function + constraints.



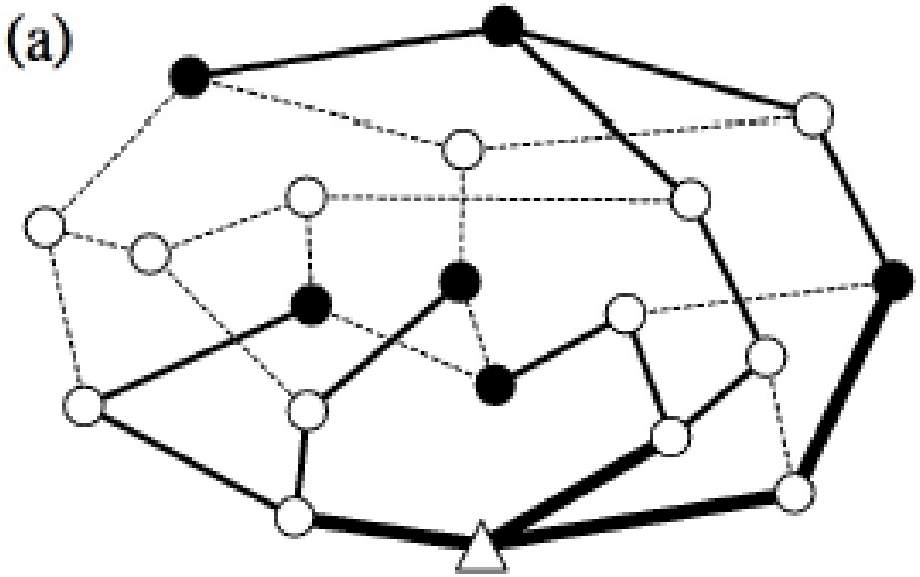
- **Factor graphs:** variable nodes and factor nodes.

- **Methods:** message-passing algorithms (cavity equations,BP,...), replica method.

- **Numerics:** C++, population dynamics, single graphs instances.



Resource allocation problem.

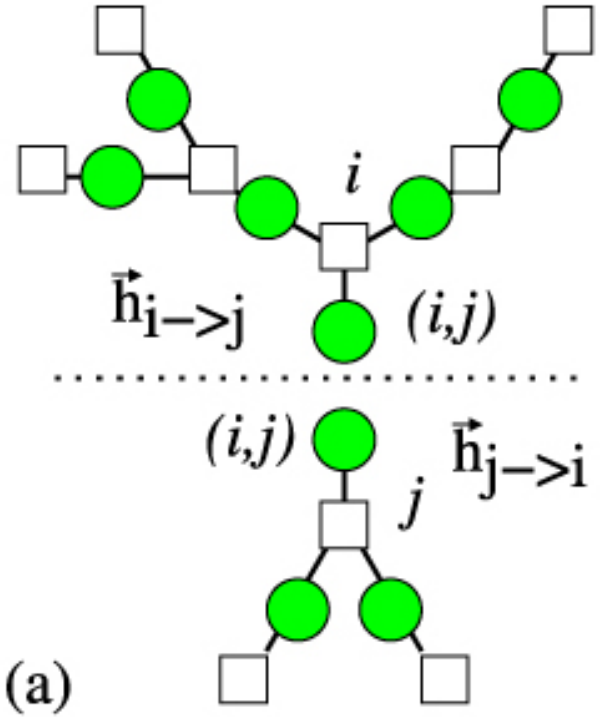


Constraint = Kirchhoff conservation law.

Black dotes = Senders (n=6)
Triangle = Receiver (n=1)

Overlap penalizing link **cost function**:

$$C(I_{ij}) := |I_{ij}|^{\alpha}$$

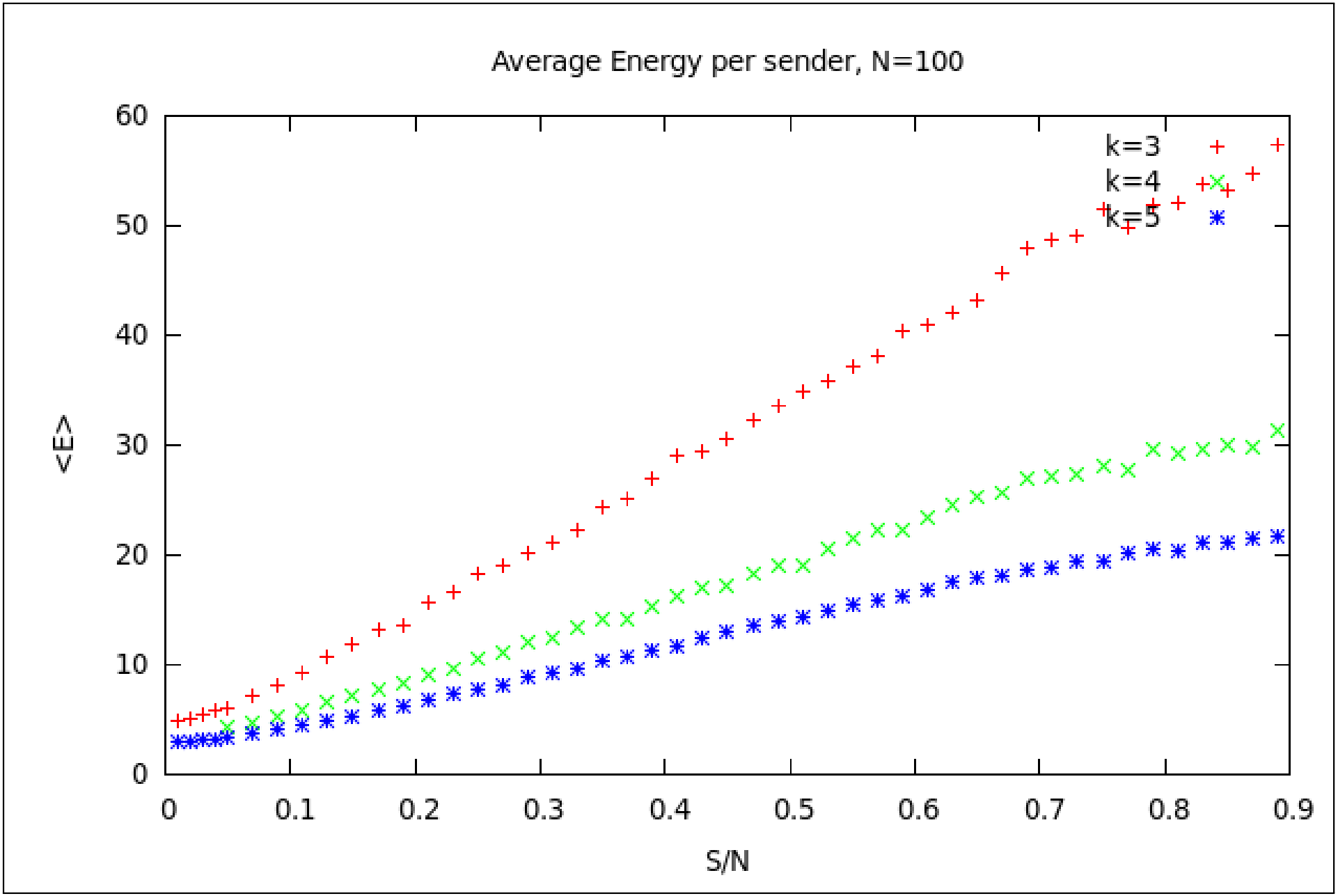


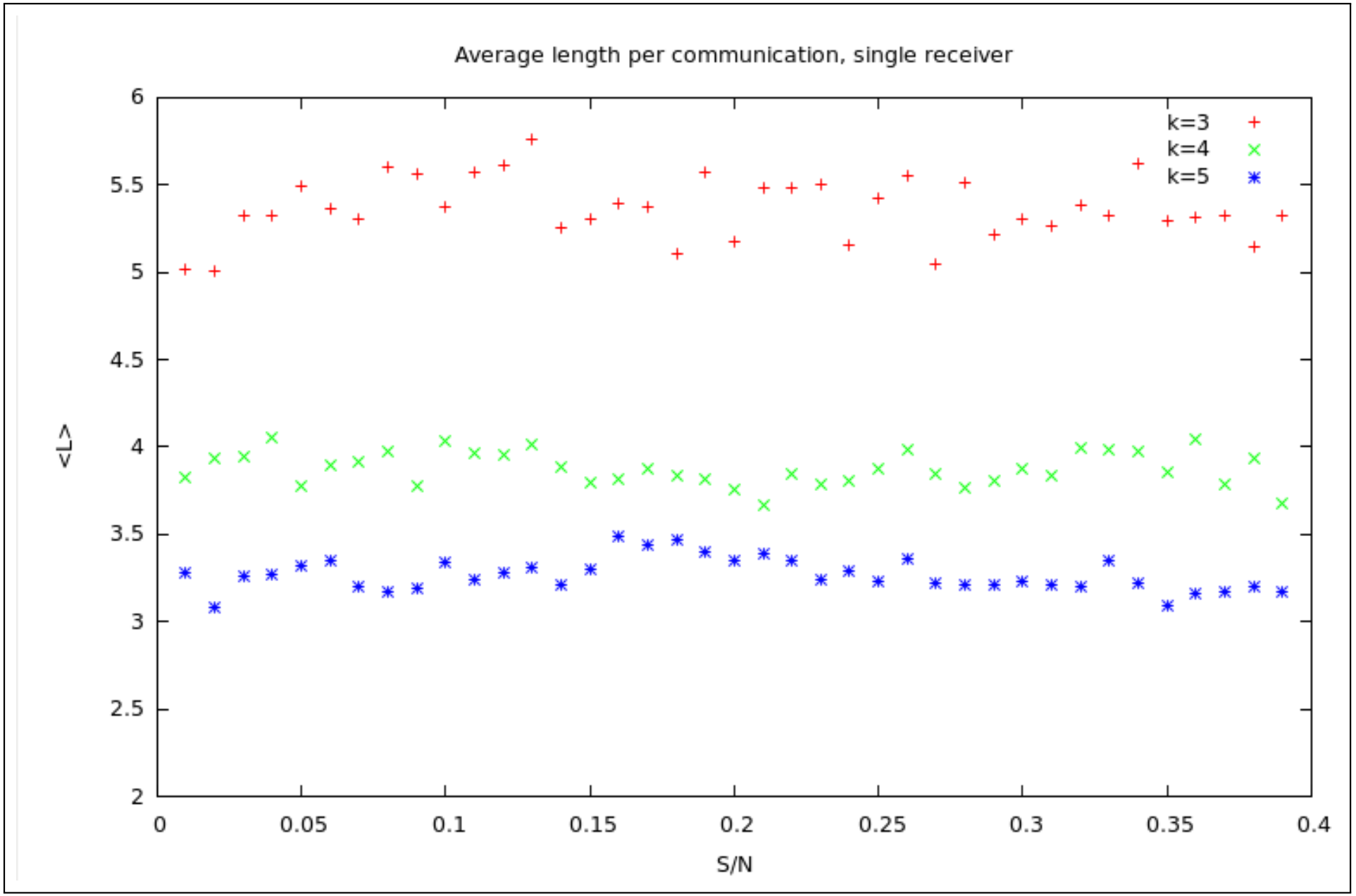
Message sent from i to j:

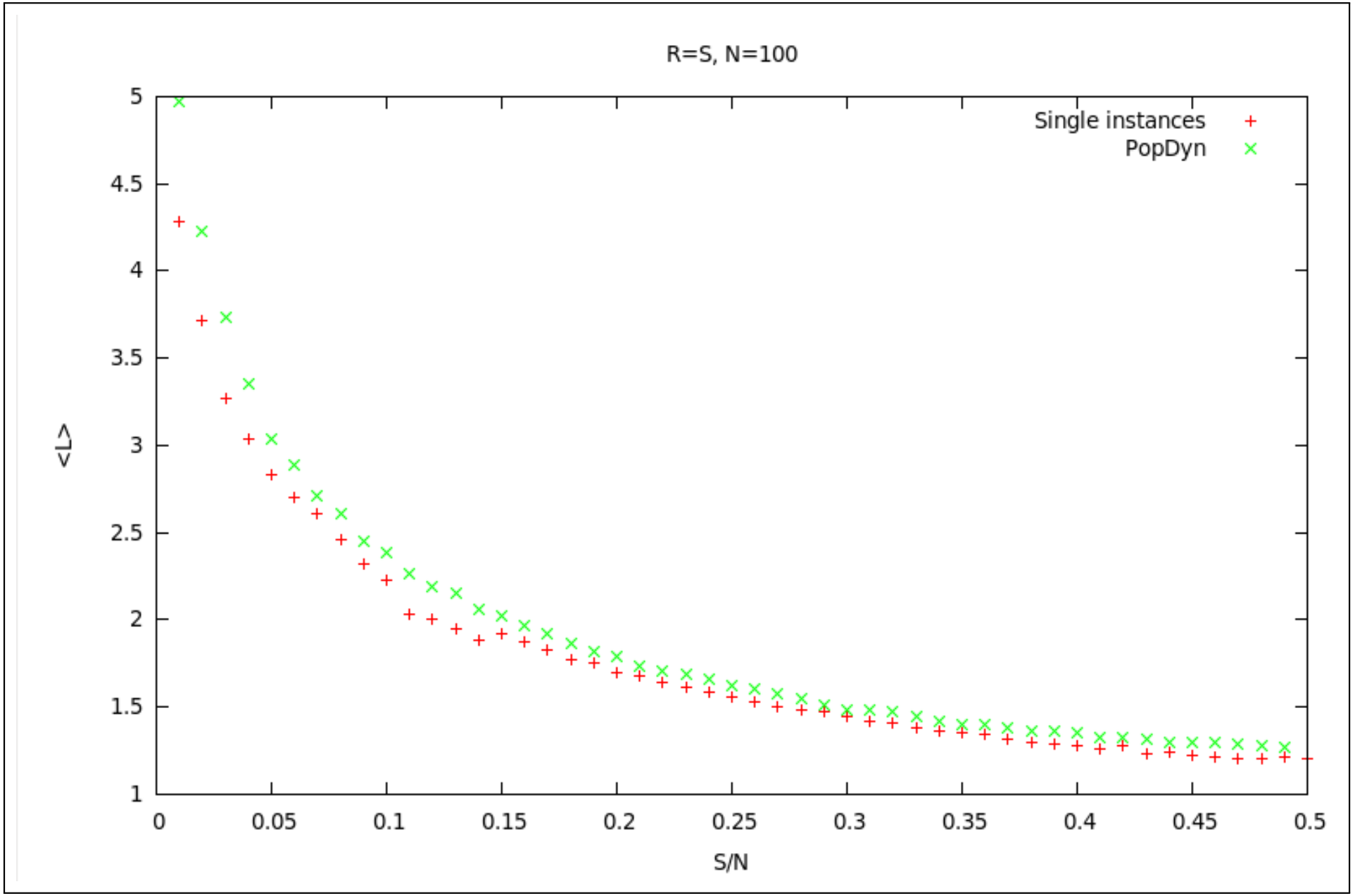
$$E_{ij}^{(t+1)}(I_{ij}) = \min_{I_{ki}|R_i=0} [|I_{ij}|^{\alpha} + \sum_{k \neq j \epsilon N_i} E_{ki}^{(t)}(I_{ki})]$$

[1]C. H. Yeung and David Saad, PRL 108, 208701 (2012).

[2] C.H. Yeung and K. Y. M. Wong, JSM P04017 (2010).


Average Energy per sender, N=100
k=3
k=4
k=5
<E>
S/N
0
10
20
30
40
50
60
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


Average length per communication, single receiver
k=3
k=4
k=5
<L>
S/N
6
5.5
5
4.5
4
3.5
3
2.5
2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4


R=S, N=100
Single instances
PopDyn
<L>
S/N
5
4.5
4
3.5
3
2.5
2
1.5
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

Improving the model:

$$\text{M communications } (S_a, R_a)$$

Each sender is assigned to one specific receiver.

Message sent from i to j:

$$E_{ij}^{(t+1)}(\bar{I}_{ij}) = \min_{\bar{I_{ki}}|R_i=0} [|\bar{I}_{ij}|^{\alpha} + \sum_{k\neq j \epsilon N_i} E_{ki}^{(t)}(\bar{I}_{ki})]$$

Possible RSB transition, phase transition...

[3] C.H. Yeung, D. Saad and K. Y. M. Wong, "From the Physics of Interacting Polymers to Optimizing Routes on the London Underground", submitted (2012)

First results.

- **Built algorithms and performed numerical simulations:** to perform BP iterations via population dynamics and single instances.
- **Reproduce Saad et al. results:** to test the validity of the algorithms.
- **Acquire knowledge on the subject:** read papers, set a bibliography, attended seminars ... .
- **Set up variations of the model:** adapt the model to different situations both theoretically and numerically.

Future implementations.

- 1. **Counting polymers**. Number of not crossing paths for a given graph.
- 2. **Planted solution** in the polymers problem of 1. To check optimality of the previous algorithm.
- 3. **Network design** consideration. Binary variables attached to edges to indicate existence of it. Introduce cost of constructing an edge.
- 4. **Sampling energy** landscapes with metastable states. Sampling with belief propagation at the metastable steps, complemented by jumps between metastable states on the basis of [4].

References:

[1] C.H. Yeung, D. Saad and K. Y. M. Wong, submitted (2012)

[2] K.Y.M. Wong and D. Saad, Phys. Rev. E 76, 011115 (2007)

[3] C.H. Yeung and K. Y. M. Wong, JSM P04017 (2010)

[4]S. Franz and G. Parisi, arXiv:1206.4067 (2012).

Secondements

- Visiting prof. **D. Saad** at Aston, Birmingham.

- Visiting prof. **R. Kuehn** at King's college.

- Combinatorial optimization at **Politecnico di Torino / Collegio Carlo Alberto** .


Thanks