Alberto Guggiola
École Normale Supérieure - Paris

Epidemic processes on networks, viral marketing and optimal vaccination

with G. Semerjian

ENS

MARIE CURIE ACTIONS

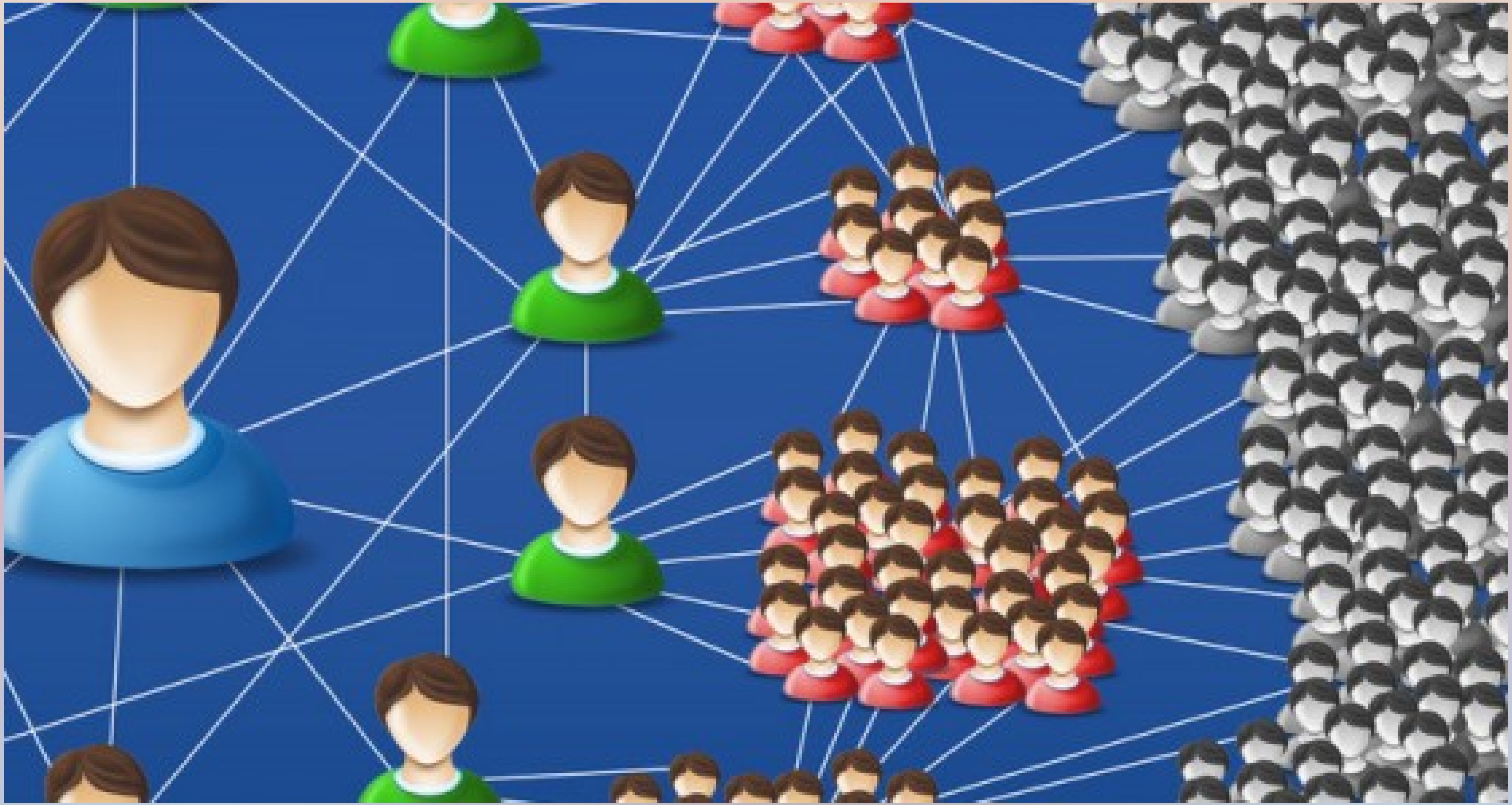
NETADIS
Statistical Physics Approaches to Networks Across Disciplines

How do we favour or inhibit the spreading of something in a given system?

Cascade failures in power grids...

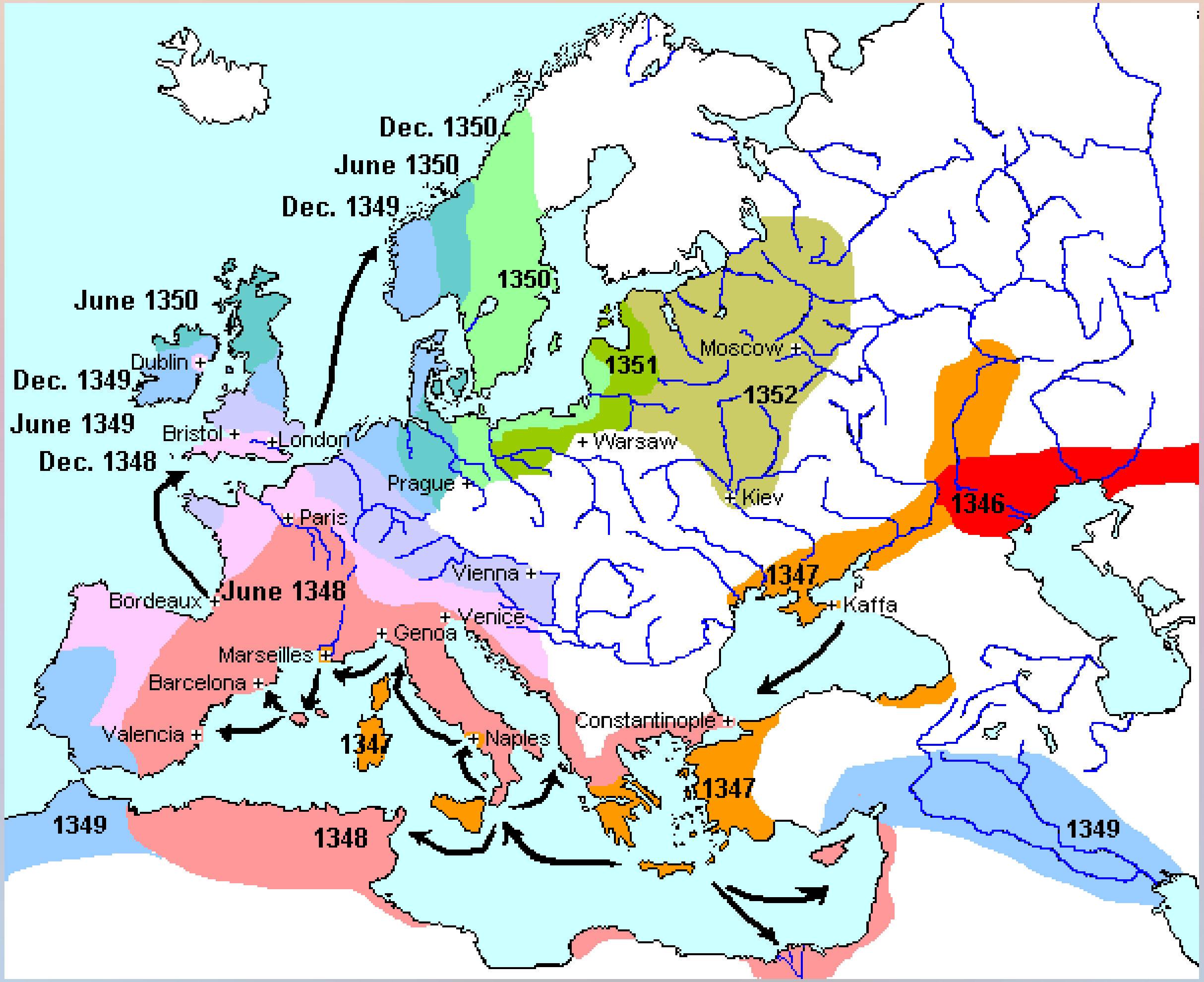
How do we favour or inhibit the spreading of something in a given system?

… effectiveness of viral marketing campaigns...



How do we favour or inhibit the spreading of something in a given system?

… diffusion of epidemics...



How do we favour or inhibit the spreading of something in a given system?

… shocks in financial markets...

Fundamental characteristics:

System:

Elements

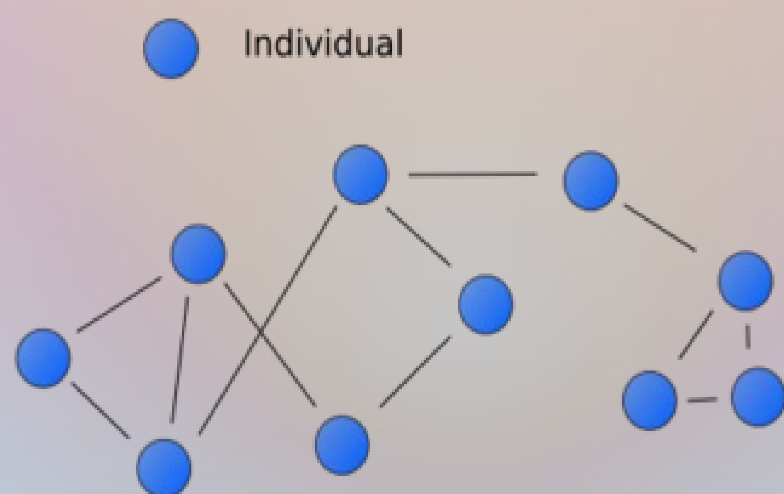
Interconnections

Fundamental characteristics:

System: | **Network:**

Elements → Nodes

Interconnections → Links



The Susceptible – Infective Model

- Two discrete states *(Compartmental Model)*

- Microscopically irreversible dynamics (no recovery) → **Short Time Processes**

A Barrat, M Barthélemy, A Vespignani, *Dynamical Processes on Complex Networks,* Cambridge University Press (2008)

The Dynamics

A node can be activated

From the beginning:
Seed of the infection

At time t:
At least l active neighbors
at time t - 1

D Kempe, J M Kleinberg, E Tardos, *Maximizing the spread of influence through a social network,* KDD 2003.

The Dynamics

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$$\Psi_i = \mathbf{1}[t_i = 0] + \mathbf{1}[t_i = f(\{t_j\})] = 1$$

D Kempe, J M Kleinberg, E Tardos, *Maximizing the spread of influence through a social network,* KDD 2003.

Formalizing the objective

Different questions...

- How many infected at most with n seeds?
- How many seeds at least to activate everyone?
- Is everyone activated after enough time?
- If not, what percentage is not touched?

Formalizing the objective

… (almost) the same answer:

Minimization of: $E(\boldsymbol{t}) = \sum E_i(t_i)$

where $E_i(t_i) = \mu \mathbf{1}[t_i = 0] + \varepsilon \mathbf{1}[t_i = \infty]$

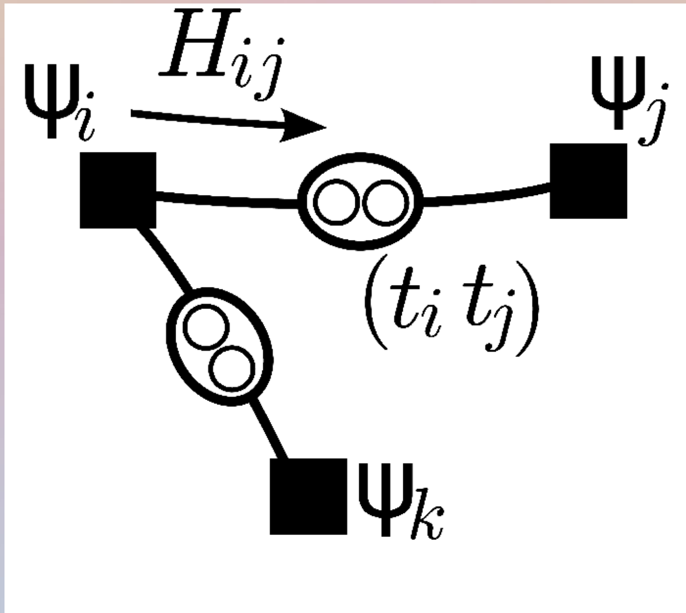
F Altarelli, A Braunstein, L Dall'Asta, R Zecchina, *Optimizing spread dynamics on graphs by message passing,* J Stat Mech (2013)

The Message Passing Algorithm

Efficient representation of the variables and the constraints of the problem

Constraints: $\Psi_i = 1, \quad \forall i$

Variables: (t_i, t_j) If nodes i and j are linked together



Consistent activation times in absence of a neighbor $H_{ij}(t_i, t_j)$ are searched iteratively

As the procedure converges, the distribution of the true activation times can be read

Two Simpler Algorithms

- **Random Choice**

New randomly chosen seeds are added until a configuration activating all the network is reached.

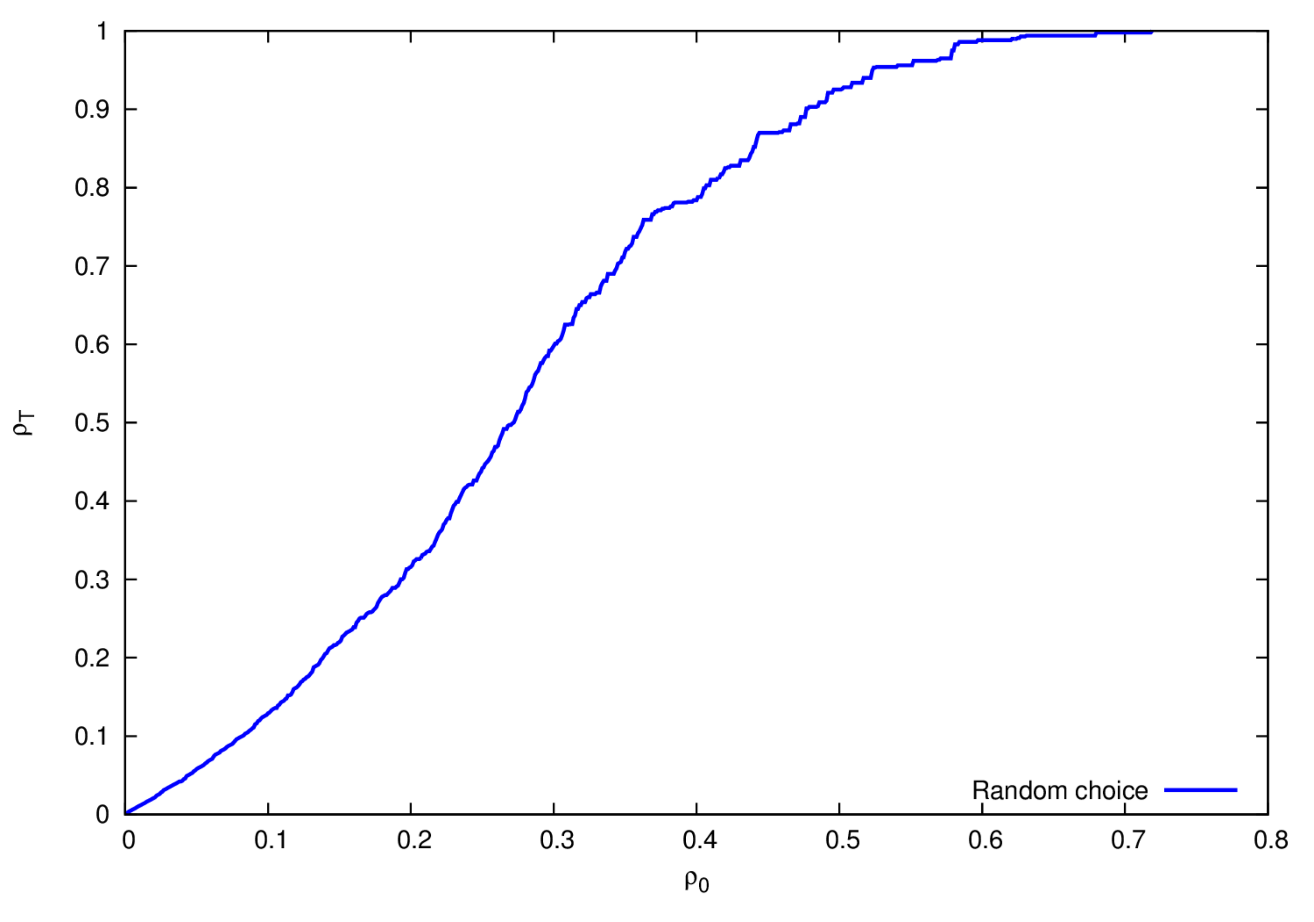
- **Greedy Strategy**

Each extra seed is chosen so to increment the most the number of activated nodes w.r.t. the previous seed configuration.

Initial vs final density

N=1000, k=3, l=2, T=5

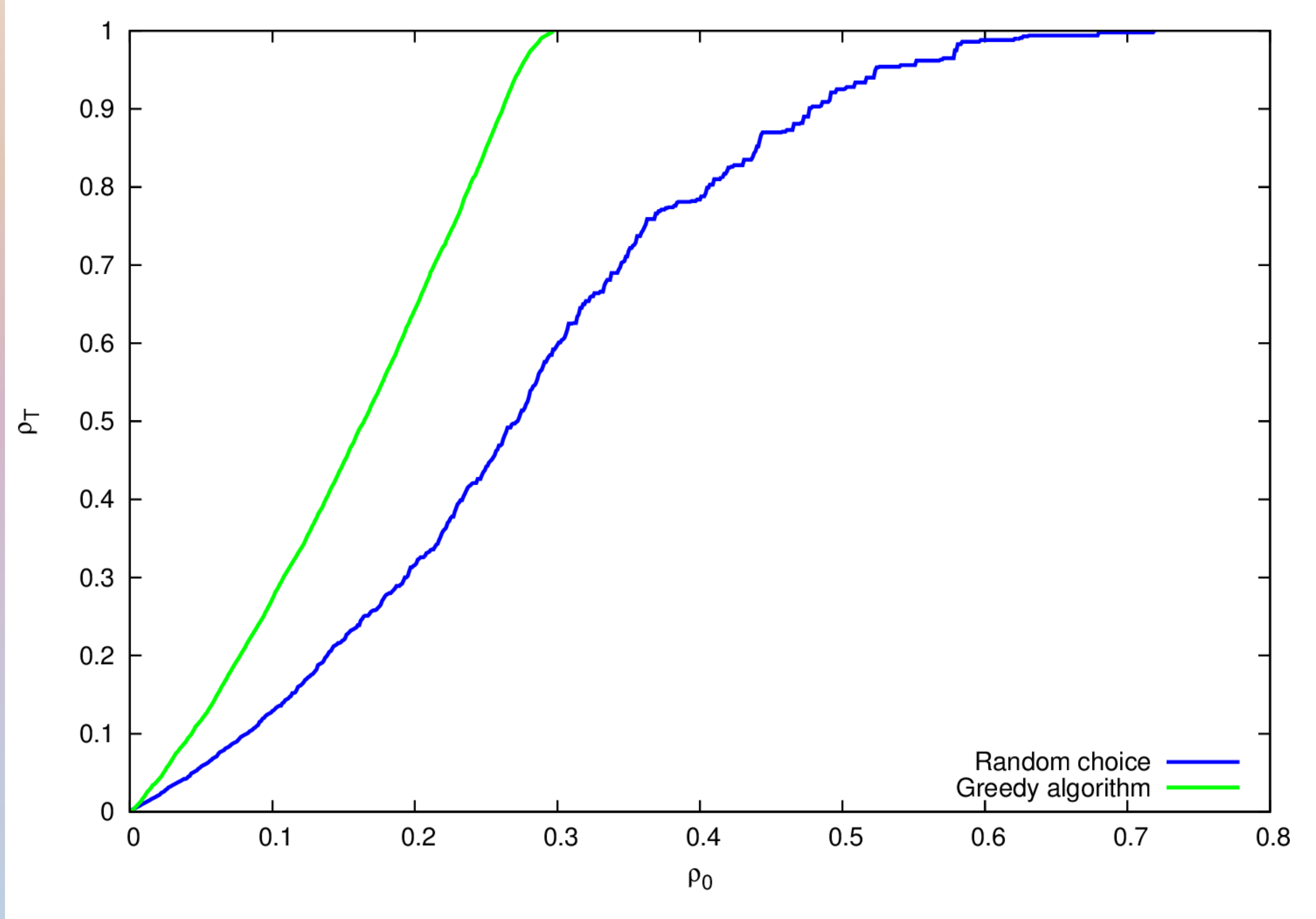
Random choice



Initial vs final density

N=1000, k=3, l=2, T=5

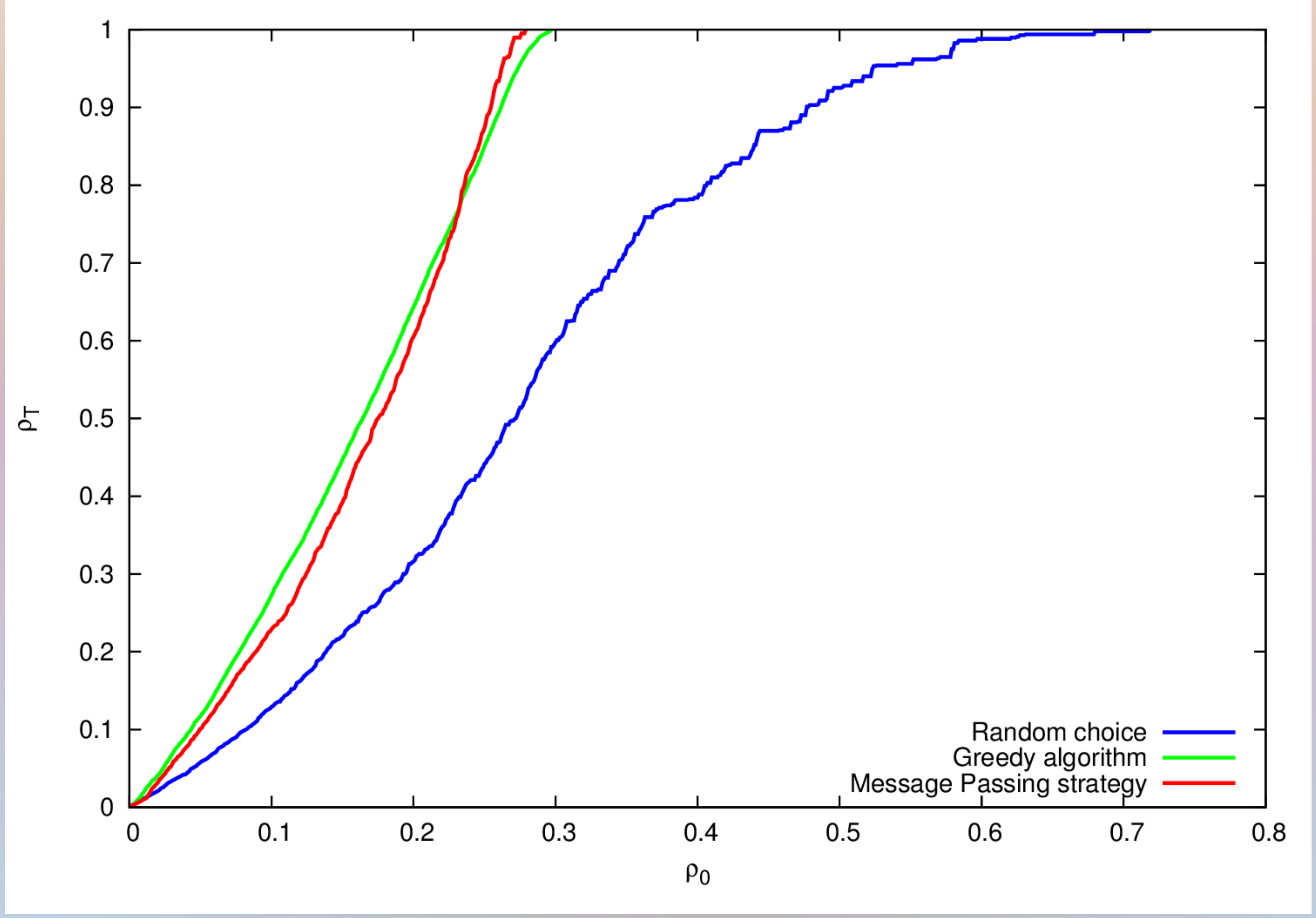
Random choice **Greedy algorithm**



Initial vs final density

N=1000, k=3, l=2, T=5

Random choice **Greedy algorithm** **MP strategy***



* 1 step RSB

Some analytical results

Seeds needed for complete activation in an infinite time: **Random vs optimal choice**

$$\theta_{optimal} = \frac{k-2}{2(k-1)} = \frac{1}{2}\theta_{random}$$

For a k – Random Regular Graph

$l = k-1$; $T \to \infty$; $N \to \infty$

Perspectives

Application to different types of graphs

Application to real world networks (e.g. biological, economical, infrastructural)

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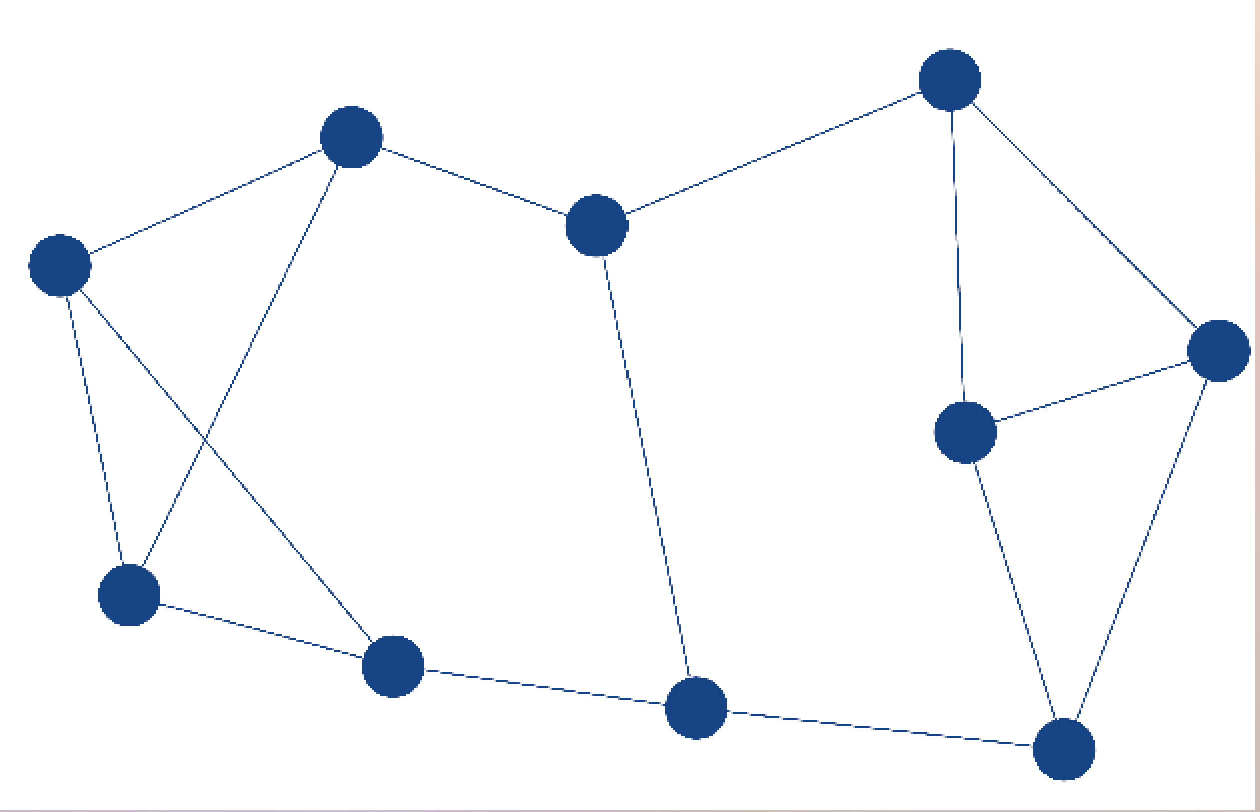
Thank you for your attention!

Random Regular Graphs



RRG with N = 10 nodes and k = 3

Random Regular Graphs

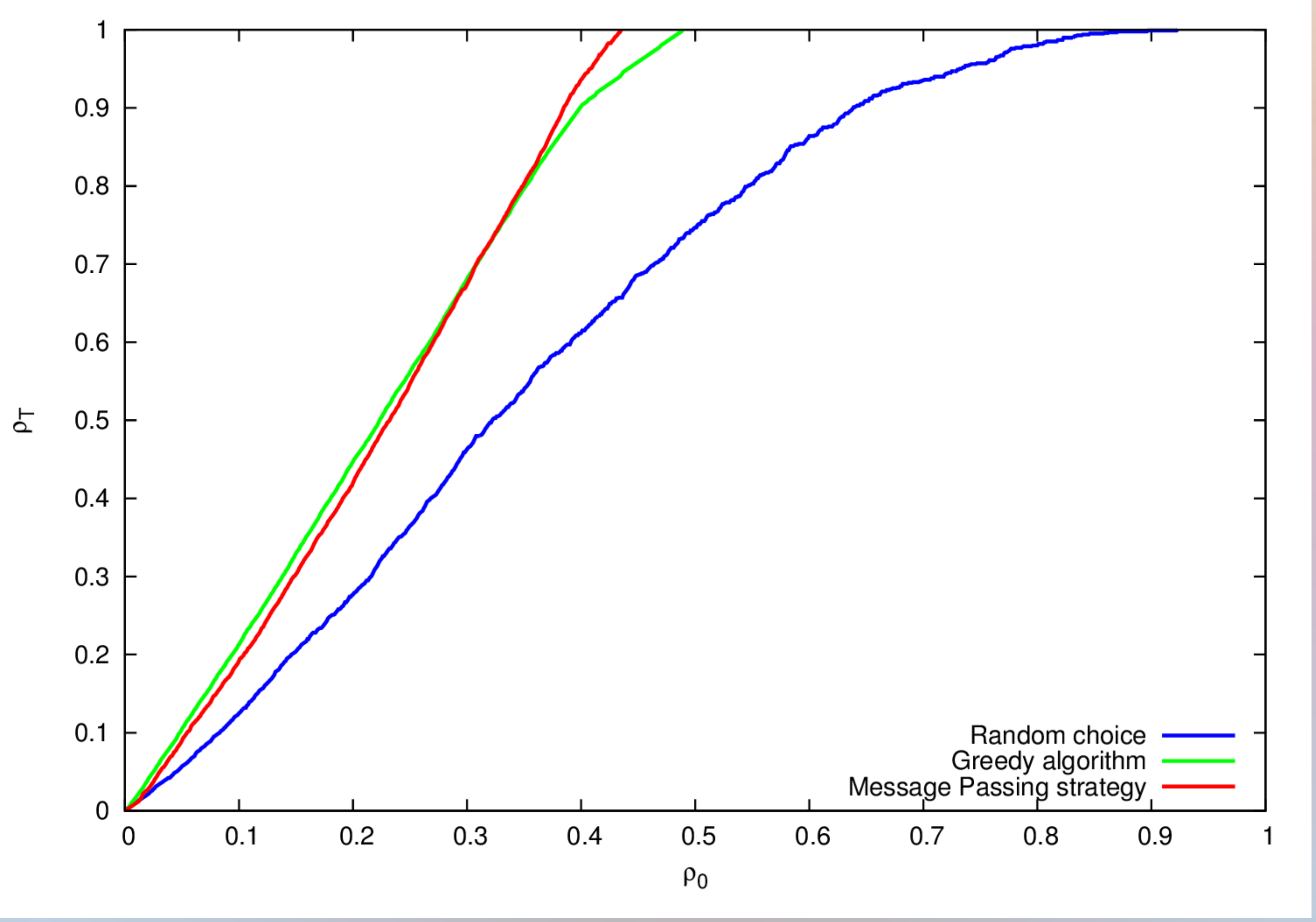
Each node has k neighbours, randomly chosen

R Albert, AL Barabàsi, *Statistical Mechanics of Complex Networks,*
Rev. Mod. Phys. 74, 47–97 (2002)

Initial vs final density

N=1000, k=3, l=2, T=1

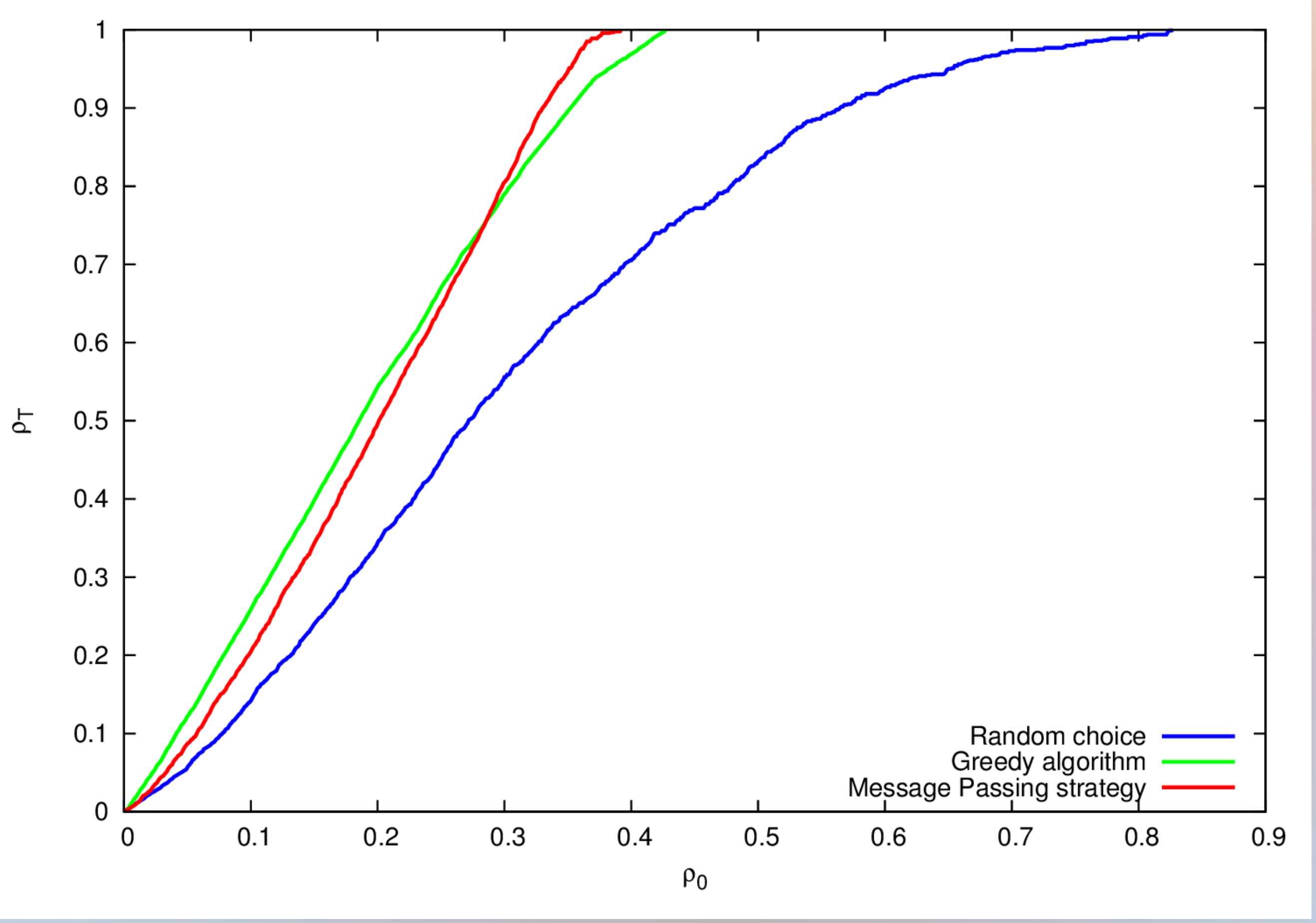
Random choice **Greedy algorithm** **MP strategy**



Initial vs final density

N=1000, k=4, l=2, T=1

Random choice **Greedy algorithm** **MP strategy**



Initial vs final density

N=800, k=3, l=2, T=5

Random choice **Greedy algorithm** **MP strategy**